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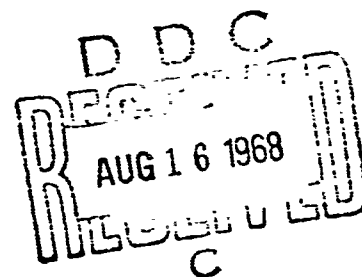
**AN INTEGRATED SIX-DEGREE-OF-FREEDOM  
TRAJECTORY AND AERODYNAMIC HEATING  
AND ABLATION COMPUTER PROGRAM**

**(STRAB-6)**

**W. J. Moulds**

**TECHNICAL REPORT NO. AFWL-TR-68-61**

**August 1968**



**AIR FORCE WEAPONS LABORATORY  
Air Force Systems Command  
Kirtland Air Force Base  
New Mexico**

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FOREWORD


This research was performed under Program Element 6.24.05.06.F, Project 5791, Task 579122.


Inclusive dates of research were July 1966 through November 1967. This report was submitted by the Air Force Weapons Laboratory Project Officer, Mr. W. J. Moulds (WLDE), on 13 May 1968.

The author wishes to express his appreciation and thanks to Lt Colonel P. D. Cronquist and Mr. C. E. O'Haver for suggesting and encouraging the accomplishment of this work. Special thanks are due to Captain J. D. Young and Lt R. H. Aungier for their assistance and answers to the numerous questions by the author. Last, but not least, the author wishes to acknowledge the assistance of Mr. J. L. Hitchcock, who did much of the programing of STRAB-6.

This technical report has been reviewed and is approved.

  
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ABSTRACT

(Distribution Limitation Statement No. 2)

An integrated computer program (STRAB-6) for the complete analysis of a blunt, conical reentry vehicle as to its trajectory and aerothermal environment upon atmospheric reentry is presented. The trajectory portion of the program calculates the vehicle motions in six-degrees-of-freedom, while the thermal portion calculates the aerodynamic heating and ablation of the vehicle. The earth model is an oblate, rotating spheroid whose parameters and gravitational potential are described herein. The equations of motion, the method of integration, thermal model and input forms for the program are discussed. The thermal model examined in this report assumes thick skin solution where the skin is of any composite structure made of discrete layers of material whose properties may vary from layer to layer. Also, the heatshield is comprised of one, two, or more different ablative materials. STRAB-6 computes nose-blunting and includes this effect in the aerodynamics. STRAB-6 is written in FORTRAN IV for the CDC 6600 computer.

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## SECTION I

## INTRODUCTION

The effects of aerodynamic heating and ablation on the motions, aerodynamic forces, and trajectory deviations of a reentry vehicle have been the subject of many analyses in recent years. Previous trajectory and heating programs have been separate, requiring that each be run separately with data from one used as input to the other. Such a method sometimes requires numerous runs before the output data converge to reasonable answers, and is both expensive in computer costs and time consuming. This report presents a computer program (STRAB-6) for the complete analysis of a reentry vehicle as to its trajectory and aerothermal environment. STRAB-6 provides a six-degree-of-freedom trajectory analysis as well as aerodynamic heating and ablation analysis for conical vehicles reentering the atmosphere at hypersonic speeds.

STRAB-6 uses some of the features from other 6-D programs (Refs. 1, 2, 3, 4, 5, 6). The trajectory portion of the program computes the six-degree-of-freedom rigid body motions for an aerodynamically symmetric reentry vehicle encountering a model atmosphere. Slender-body aerodynamic theory was used and the required aerodynamic coefficients are provided by subroutine input (as a function vehicle shape, angle-of-attack, and either Mach number or altitude). The atmospheric model that is used is based on the 1962 ARDC atmosphere, and the geocentric model that is used is a rotating oblate spheroid.

The aerodynamic heating and ablation portion of the program (Ref. 7) (HEATAB), in its original form, computed point mass heating for a sharp-nosed conical vehicle. The basic heating equations of this original program (HEATAB) are described in reference 7. There have been numerous improvements to the original program to include: heating on a blunt-nosed vehicle, the case where a vehicle heatshield is comprised of two or more different ablators, and boundary layer edge conditions.

The equations of motion and of heat transfer are general in that they apply to most axially symmetric vehicles. Even though this program considers the body in general motion under combined angles of attack and sideslip, the vehicle is assumed to ablate symmetrically. The ablation weight loss, nose bluntness, and vehicle volume changes are considered in the calculations for

the viscid and inviscid drag coefficients ( $C_D$ ) and stability derivatives, as well as the vehicle mass moment of inertia properties. In addition, the aerodynamic coefficients are calculated for the complete range of angle-of-attack ( $0 \leq \alpha \leq 360^\circ$ ) and sideslip ( $0 \leq \beta \leq 360^\circ$ ).

The position vector is calculated in a rotating earth-centered coordinate system, while the angular positions are calculated in a reentry-centered coordinate system. Computations are carried out, using the Runge-Kutta method of numerical integration employing a fixed step size mode.

This program is specific in that the equations have been applied to a blunt-nosed conical vehicle with the starting point at reentry conditions. With some modifications, the user could easily adapt the program to analyze a missile from launch by adding thrust subroutines and substituting launch conditions for reentry conditions.

## SECTION II

## MAIN PROGRAM DESCRIPTION

## 1. GENERAL

The computer program, STRAB-6, is a self-contained program in the sense that it can generate the required aerodynamic data, and calculate the mass and inertial properties due to ablation changes of a symmetrical vehicle without any additional preliminary calculations.

STRAB-6 calculates the resultant loads, motions, and trajectory of a rigid vehicle in general 6-D motion as well as boundary-layer properties and time-temperature distribution in a body. If a nonablating trajectory is desired, an option is provided to omit the heating and ablation portion of the program.

At the initiation of a reentry run, the vehicle is located in space with the desired reentry conditions (i.e., velocity, flight path angle, altitude, roll rate). Also, the user must input the latitude, longitude, and azimuth at which he wishes to start the trajectory. Other trajectory parameters the user may wish to input are angle-of-attack and angle-of-sideslip; otherwise, these values are zero for a nominal trajectory.

The aerodynamic drag coefficients (CD) are obtained from Newtonian slender-body theory for inviscid drag and modified Blasius flat plate theory for viscous drag. These coefficients are calculated in subroutines CDSF and CDFM and are limited to vehicle shapes of spherically blunted cones where the ratio of base radius to axial length is approximately equal to the cone half-angle. A discussion of these theories is in section II-5.

The atmospheric model used in the program is based on the 1962 ARDC model atmosphere, and it expresses the density, ambient temperature, pressure, and the speed of sound as functions of altitude. The geocentric model used is a rotating oblate spheroid, where the sea level radius,  $R_E$ , is (figure 1)

$$R_E = \frac{R_{EQ}}{\left(1 + P_E \sin^2 \phi_c\right)^{1/2}} \quad (1)$$

where

$R_{EQ}$  = earth equatorial radius-ft

$\lambda_c$  = geocentric latitude-degree

$$P_E = \frac{e_E^2}{1 - e_E^2} = 0.00673852 \quad (\text{Ref 8}) \quad (2)$$

$e_E$  = eccentricity of earth = 0.0818133302

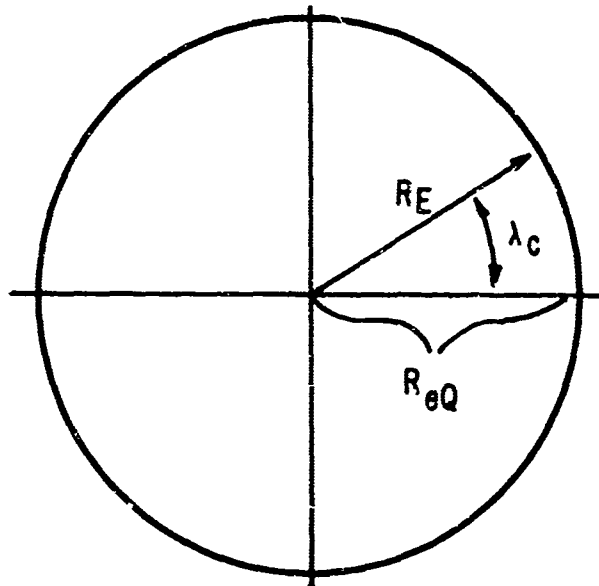


Figure 1. Geocentric Earth Model

The vehicle thermal model as presented in HEATAB has been modified to extend the calculations to include the total integrated ablation mass loss rate for a conical reentry vehicle. The body is divided into a number of segments, or stations along the cone, and the heating rate at each station is computed. In addition to the original input data required by HEATAB, the fore and aft radii, slant length, and transition Reynolds number of each body segment must be supplied. These input data are identified as: Data RS, Data RL, Data SOL, and Data RN. Computations of the skin temperature profile and ablation mass loss rate are identical to the method in HEATAB, except that these calculations are made for each X-station at each trajectory point.

As written, the STRAB-6 output will give, for each time interval, an output of trajectory conditions, body conditions, and aerodynamic parameters. The trajectory output conditions include time, altitude, velocity, and flight path angle. The body conditions for each X-station include weight loss rate per unit area, wall recession, and wall temperature profile. The aerodynamic parameters include CD, maximum aerodynamic load, angle-of-attack and sideslip, restoring forces and moments, and boundary layer edge conditions.

## 2. EQUATIONS OF MOTION

The equations of motion for an ascent or descent through the atmosphere can be written with various degrees of complexity. The equations differ in form depending upon the coordinate system, the number of degrees of freedom, the method of adjusting for the earth's rotation, and the integration scheme. In the evaluation of reentry vehicles for height-of-burst (HOB) errors, dispersion analysis, or circular error probable (CEP), the equations of motion must be written with six-degrees-of-freedom (6-D). This program employs a Runge-Kutta method of integration for a high degree of accuracy.

### a. Coordinate Systems

The equations of motion can be written in vector notation using Newton's second law. A summation of the forces acting on the vehicle results in the equation

$$m \frac{d^2 \vec{R}}{dt^2} = \sum \vec{F} - 2m\vec{\omega} \times \vec{v} - m\vec{\omega} \times (\vec{\omega} \times \vec{R})$$

Before this equation can be solved, however, it must be expressed in terms of a convenient coordinate system.

There are five principal coordinate systems or reference frames associated with orbital mechanics and trajectories. Various other reference frames can be used depending on the complexity of the problem to be solved. For this report, there are three reference frames: inertial reference frame, topocentric or horizontal reference frame, and earth reference frame. The following is a description of each and their interrelationships. All the reference frames are right-handed orthogonal coordinate systems.

## (1) Inertial Reference Frame

The basic coordinate for the program is a nonrotating inertial reference frame fixed with its origin at the geocenter as shown in figure 2.  $Z_R$  points north along the axis of rotation of the earth and  $X_R$  and  $Y_R$  lie in the equatorial plane.  $X_R$  lies in the prime meridian plane at the start of a trajectory run (time,  $t = 0$ ). The earth rotates with an angular velocity  $\omega_E$  about  $Z_R$ ; hence the earth reference frame, E, differs from the inertial frame, R, by a rotation  $\omega_E t$ .

The vehicle center of gravity (CG) is located in the inertial R frame by the longitude,  $\beta$ , latitude,  $\lambda_c$ , and vehicle altitude, ALT. The distance from the geocenter to the vehicle CG, measured along the vector whose orientation is determined by the angles  $\beta$  and  $\lambda_c$  is

$$R = R_E + \text{ALT} \quad (3)$$

where  $R_E$  is defined by equation (1).

It may be worthwhile to digress briefly on the subject of latitude. There are three latitudes commonly used in astrodynamics; however, in this report we will be concerned only with geocentric and geodetic. Geocentric latitude,  $\lambda_c$ , is defined as the angle, measured at the earth's center, between the equatorial plane and a radius,  $R_E$ , to the point of interest on the earth's surface. The geodetic latitude,  $\lambda_g$ , is the angle between the equatorial plane and a normal to a reference spheroidal surface,  $Z_H$ , as shown in figure 3.

The reference spheroid is an oblate ellipsoid whose surface nearly approximates the sea-level surface of the earth. Such a spheroid is characterized by a "flattening" or "oblateness,"  $f$ , where

$$f = (a - b)/a \quad (4)$$

and  $a$  is the semimajor, and  $b$  the semiminor axis of the ellipse of revolution forming the spheroid. The derivation of equation (1) can be easily obtained from the equation for an ellipse if  $a$  equals equatorial radius,  $R_{EQ}$ , and  $b$  equals polar radius,  $R_{Po}$ . Therefore,

$$\frac{x^2}{R_{EQ}^2} + \frac{z^2}{R_{Po}^2} = 1 \quad (5)$$

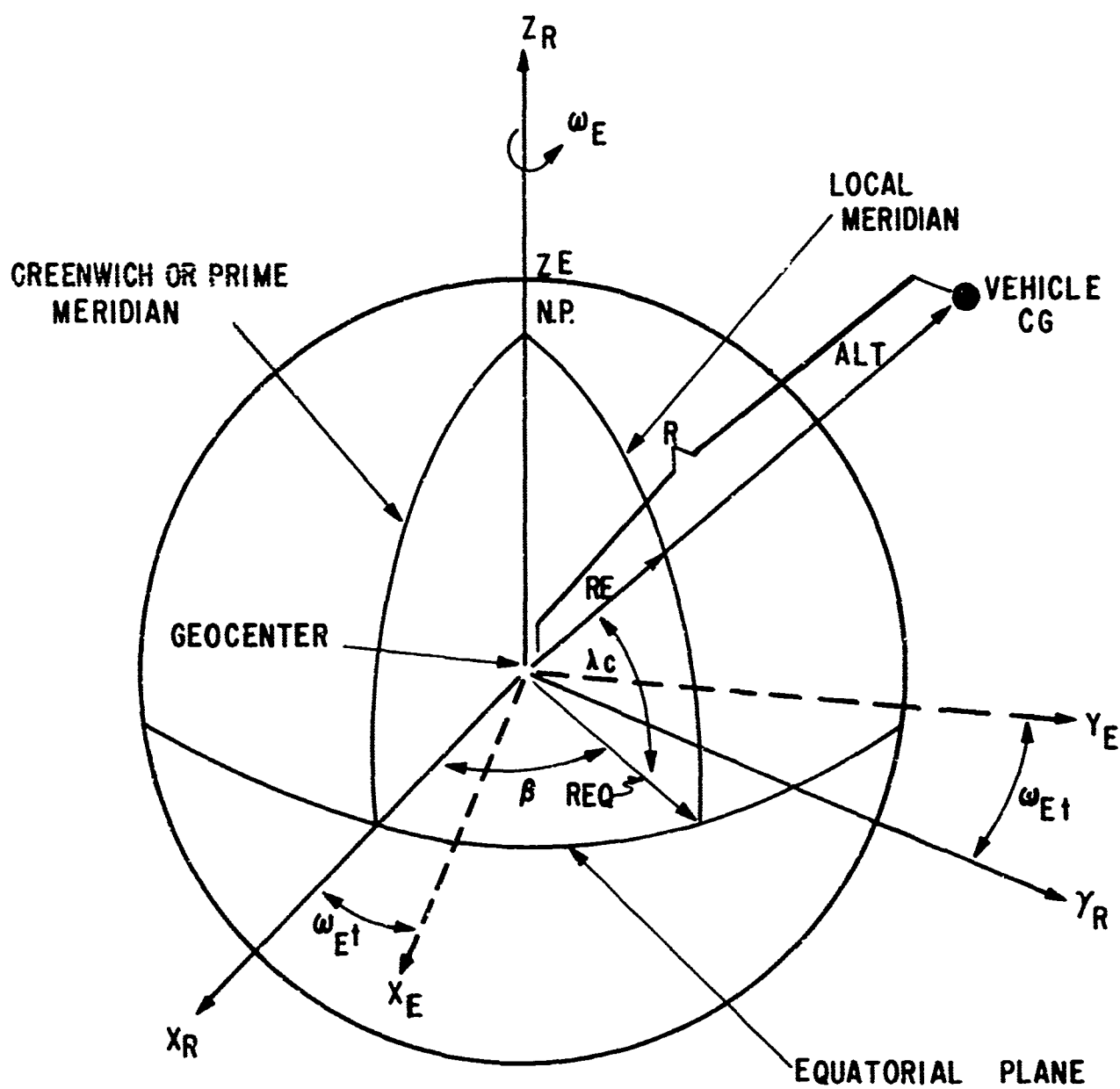


Figure 2. Inertial Reference Frame, R, and Earth Reference Frame, E



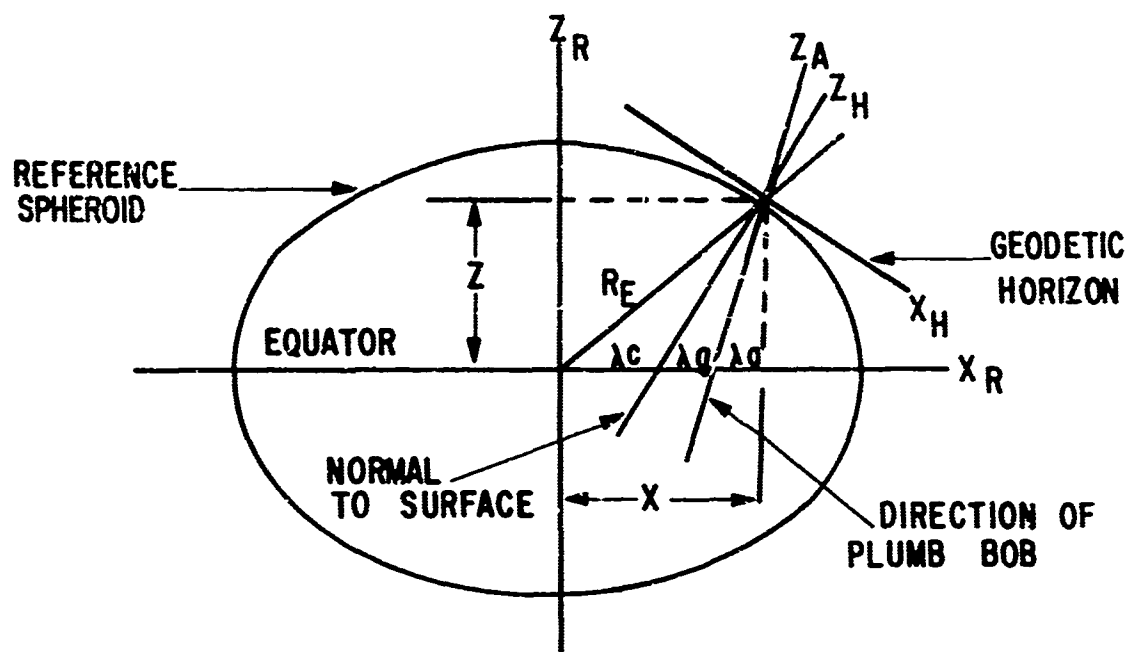


Figure 3. Geocentric, Geodetic, and Astronomical Latitudes (Ref. 9)

By multiplying  $R_{EQ}^2$  and adding and subtracting  $z^2$  on the left side,

$$x^2 + z^2 - z^2 + \frac{R_{EQ}^2 z^2}{R_{Po}^2} = R_{EQ}^2$$

But from figure 3

$$x^2 + z^2 = R_E^2$$

$$z = R_E \sin \lambda_c \quad (6)$$

Therefore,

$$R_E^2 + R_E^2 \sin^2 \lambda_c \left( \frac{R_{EQ}^2}{R_{Po}^2} - 1 \right) = R_{EQ}^2 \quad (7)$$

Solving for  $R_E$

$$R_E = \frac{R_{EQ}}{\left[ 1 + \sin^2 \lambda_c \left( \frac{R_{EQ}^2}{R_{Po}^2} - 1 \right) \right]^{1/2}} \quad (8)$$

From reference 10

$$f = 1 - \sqrt{1 - e_E^2} \quad (9)$$

and from equation (4)

$$\frac{R_{EQ}}{R_{Po}} = \frac{1}{1 - f}$$

By squaring both sides and substituting equation (9) for  $f$ ,

$$\frac{R_{EQ}^2}{R_{Po}^2} = \frac{1}{1 - e_E^2} \quad (10)$$

Now, by adding -1 to both sides,

$$\frac{R_{EQ}^2}{R_{Po}^2} - 1 = \frac{1}{1 - e_E^2} - 1 = \frac{e_E^2}{1 - e_E^2}$$

From equation (2), it is shown that

$$P_E = \frac{R_{EQ}^2}{R_{Po}^2} - 1 \quad (11)$$

Equation (1) is now obtained by substituting equation (11) into equation (8).

$$R_E = \frac{R_{EQ}}{\left( 1 + P_E \sin^2 \lambda_c \right)^{1/2}}$$

The sea-level radius of the earth can now be determined at any station by knowing the geocentric latitude,  $\lambda_c$ , of the station.

The astronomical latitude,  $\lambda_a$ , is defined as the angle between the local astronomical zenith,  $Z_a$ , or vertical (as determined by a plumb bob or, actually, by the direction of the local gravitational field) and the equatorial plane. Since astronomical latitude is based upon the local gravitational field and is also centrifugal-force affected, it differs from the geodetic latitude by a very small value and can be considered negligible. Therefore, this report will consider geodetic and astronomical latitudes identical and will be concerned only with the relationship between geodetic and geocentric.

To convert from geodetic latitude to geocentric latitude, the slope ( $\theta$ ) of the tangent (geodetic horizon, see figure 4) at the station on the earth's surface can be obtained by differentiating equation (5).

$$\frac{2x}{R_{EQ}^2} + \frac{2z}{R_{PO}^2} \frac{dz}{dx} = 0$$

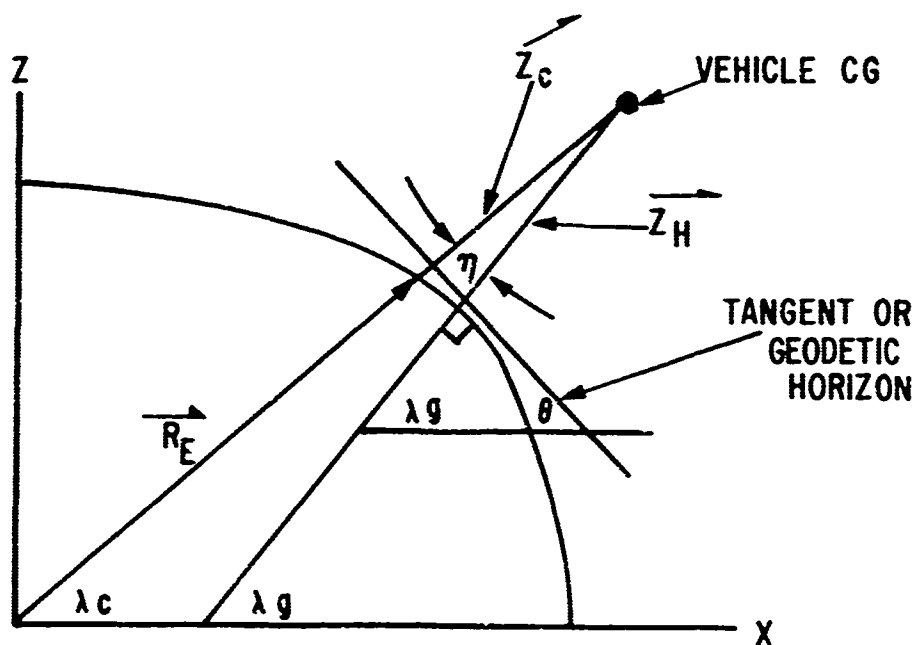


Figure 4. Geodetic to Geocentric

Therefore,

$$\tan \theta = \left| \frac{dz}{dx} \right| = \left| - \frac{R_{Po}^2}{R_{EQ}^2} \frac{x}{z} \right| \quad (12)$$

but

$$\tan \theta = \frac{1}{\tan \lambda_g}$$

$$\tan \lambda_c = \frac{z}{x}$$

By substituting these quantities into equation (12), the relationship between  $\lambda_g$  and  $\lambda_c$  is established.

$$\tan \lambda_g = \frac{R_{EQ}^2}{R_{Po}^2} \tan \lambda_c \quad (13a)$$

But by substituting equation (10) into equation (13a),  $\lambda_c$  can be determined from one constant. Therefore,

$$\lambda_c = \arctan \left[ \left( 1 - e_E^2 \right) \tan \lambda_g \right] \quad (13b)$$

$$\eta = \lambda_g - \lambda_c \quad (14)$$

With the foregoing relationships established, the second reference frame can now be defined.

## (2) Topocentric or Horizontal Reference Frame

The second of these coordinate systems is commonly known as the topocentric, but has other names depending upon the users application. Some of the names are horizon, launch, reentry, and trajectory system. This report refers to this coordinate system as the horizontal, H. The word topocentric

is derived from the Greek *topos*, meaning a place. In this reference frame, the origin is taken at the observer as shown in figure 5. The  $Y_H$ -axis is directed toward the east, the  $X_H$ -axis toward the south, and the  $Z_H$ -axis is directed toward the astronomical zenith. This system is rotated along with the earth's reference frame, and is, therefore, not inertial. The angle from the north direction measured clockwise (or east) in the  $X_H, Y_H$  plane is defined as azimuth, AZ.

With the horizontal reference frame now defined, it is now possible to complete three of the six-degrees-of-freedom.

#### b. Equation Development

Newton's second law for a rotating coordinate system is repeated here for convenience.

$$m \frac{d^2 \vec{R}}{dt^2} = \sum \vec{F} - 2m\vec{\omega} \times \vec{v} - m\vec{\omega} \times (\vec{\omega} \times \vec{R})$$

or

$$\frac{d^2 \vec{R}}{dt^2} = \frac{\sum \vec{F}}{m} - 2 \left( \vec{\omega}_E \times \frac{d\vec{R}}{dt} \right) - \vec{\omega}_E \times (\vec{\omega}_E \times \vec{R}) \quad (15)$$

where

$\frac{d^2 \vec{R}}{dt^2}$  is the vector acceleration of the vehicle CG with respect to the inertial reference frame

$\vec{F}$  is the total force vector on the reentry vehicle

$2 \left( \vec{\omega}_E \times \frac{d\vec{R}}{dt} \right)$  is the coriolis acceleration of the axis system due to the rotation of the earth

$m$  is vehicle mass

$\vec{\omega}_E \times (\vec{\omega}_E \times \vec{R})$  is the centrifugal acceleration of the axis system due to the earth's rotation

$\omega_E = 0.7292 \times 10^{-4}$  radian/sec, earth rotational angular velocity

Since the horizontal reference system  $(X_H, Y_H, Z_H)$  will rotate with the earth, several vector cross-products will appear in the final equation for  $d^2 \vec{R}/dt^2$ . Therefore, the rotation rate vector,  $\vec{\omega}_E$ , and general vector,  $\vec{R}$ , must be expressed in the same reference frame  $(X_H, Y_H, Z_H)$  for expansion.

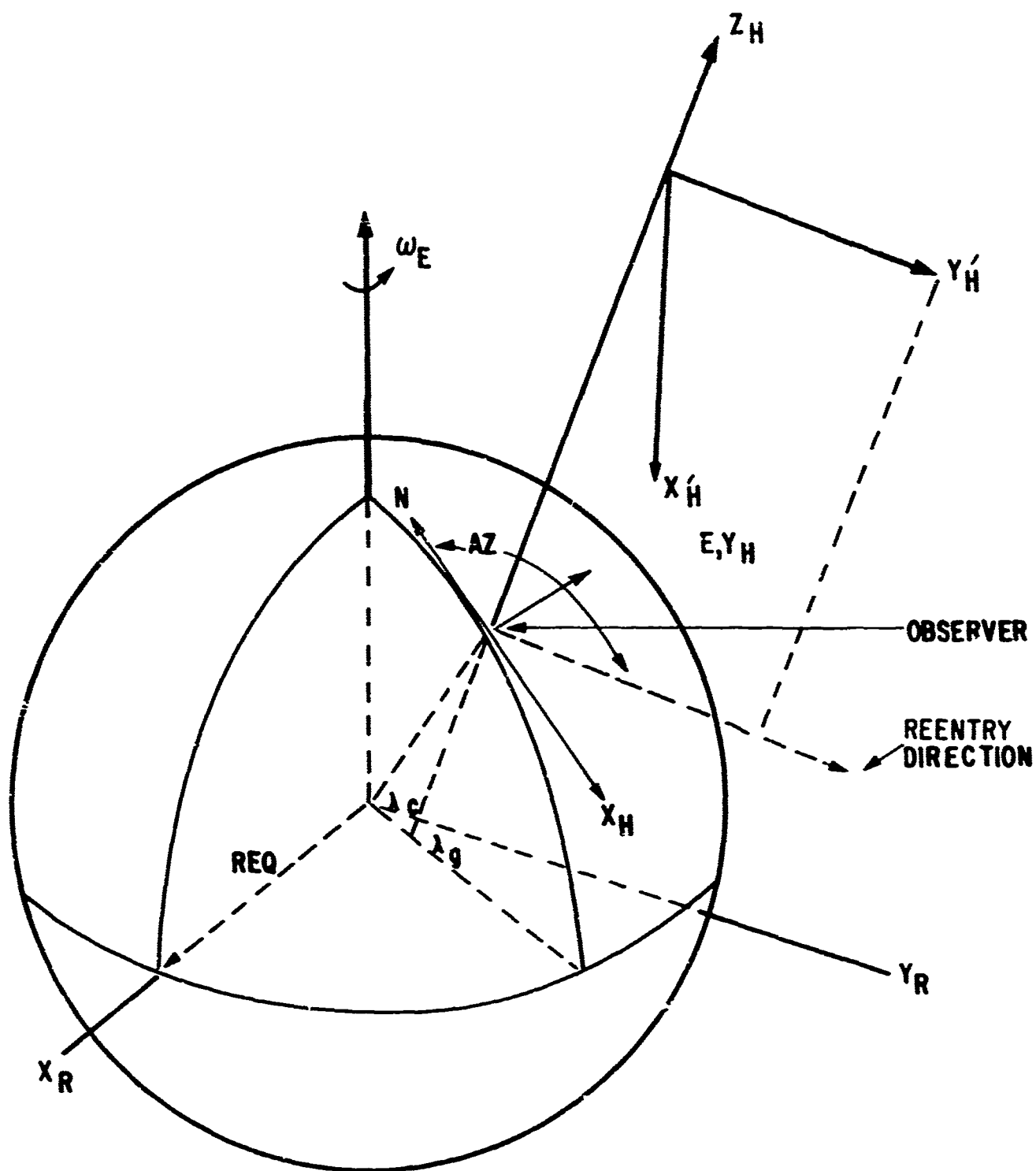


Figure 5. Horizontal Reference Frame

As shown in figure 5, pseudo-trajectory reference frame  $(X_H^i, Y_H^i, Z_H^i)$  has its origin at the initial position of the vehicle center of gravity. This frame is fixed in space (with respect to the horizontal (H) frame), with  $Z_H^i$  equal to the reentry altitude, and with  $X_H^i$  and  $Y_H^i$  rotated from  $X_H$ ,  $Y_H$  about  $Z_H$  some initial azimuth angle,  $AZ$ . The vehicle flight path is initially fixed in the  $Y_H^i - Z_H^i$  plane directed downward some reentry angle,  $\gamma_e$ .

Therefore, the components of  $X_H$ ,  $Y_H$ ,  $Z_H$  on  $X_H^i$ ,  $Y_H^i$ ,  $Z_H^i$  is

$$\vec{X}_H^i = \cos AZ \cdot \vec{X}_H + \sin AZ \cdot \vec{Y}_H \quad (16a)$$

$$\vec{Y}_H^i = \sin AZ \cdot \vec{X}_H - \cos AZ \cdot \vec{Y}_H \quad (16b)$$

$$\vec{Z}_H^i = \vec{Z}_H \quad (16c)$$

As previously shown, the horizontal reference frame is not inertial, and therefore, rotates at the same rotation rate as the earth. So that  $\vec{\omega}_E$  may be written

$$\vec{\omega}_E = \omega_E \sin \lambda_g \vec{Z}_H - \omega_E \cos \lambda_g \vec{Y}_H \quad (17)$$

Equation (17) may be rewritten after substituting equation (16b) into equation (17).

$$\begin{aligned} \vec{\omega}_E &= \omega_E \sin \lambda_g \vec{Z}_H - \omega_E \cos \lambda_g \cdot \sin AZ \vec{X}_Y \\ &\quad + \omega_E \cos \lambda_g \cdot \cos AZ \vec{Y}_H \end{aligned} \quad (18)$$

Therefore, the components of the earth's rotation rate in the horizontal reference frame may be written

$$\omega_{EX_H} = -\omega_E \cos \lambda_g \cdot \sin AZ \quad (19a)$$

$$\omega_{EY_H} = \omega_E \cos \lambda_g \cdot \cos AZ \quad (19b)$$

$$\omega_{EZ_H} = \omega_E \sin \lambda_g \quad (19c)$$

To determine the general vector,  $\vec{R}$ , and displacement accelerations, the vehicle CG must be considered displaced from the horizontal reference frame by the vector,  $\vec{R}$ , as shown in figure 6.

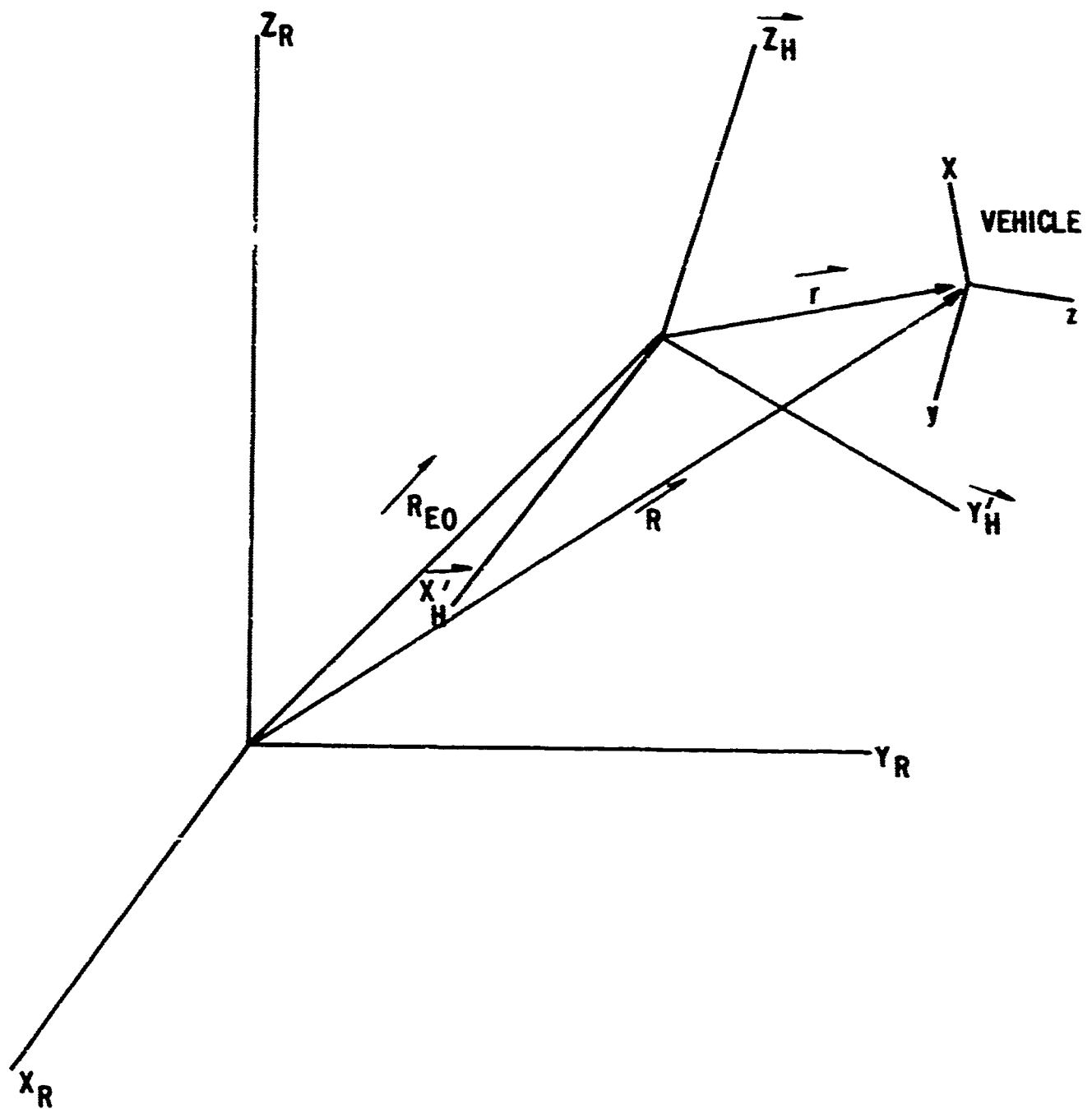


Figure 6. Inertial and Body Axis Coordinate Systems



From figure 6, the following relationship can be established:

$$\vec{R} = \vec{R}_{eo} + \vec{r} \quad (20)$$

$\vec{r}$  can be written in terms of its components on the horizontal reference frame as

$$\vec{r} = X \vec{X}'_H + Y \vec{Y}'_H + Z \vec{Z}'_H \quad (21)$$

where X, Y, Z are the scalar vehicle CG displacement on the horizontal reference frame.  $\vec{R}$  can now be written as

$$\vec{R} = X \vec{X}'_H + Y \vec{Y}'_H + Z \vec{Z}'_H + \vec{R}_{eo} \quad (22)$$

Although  $\vec{R}$  can be written in any reference frame, its vector components must be in the same reference frame. Therefore, the equation

$$\vec{R}_{eo} = R_{eo} \vec{Z}_c \quad (23)$$

must be written in terms of the horizontal reference system.

From figure 4, the vector,  $\vec{Z}_c$ , in the geocentric reference frame can be written in the form

$$\vec{Z}_c = \vec{Z}_H \cos \eta + \vec{Y}_H \sin \eta$$

or by substituting in equations (16b) and (16c), the following is obtained:

$$\vec{Z}_c = \vec{Z}_H \cos \eta + \vec{X}'_H \sin \eta \sin AZ - \vec{Y}'_H \cos \eta \sin AZ \quad (24)$$

Equation (23) can now be rewritten as

$$\vec{R}_{eo} = R_{eo} \sin AZ \sin \eta \vec{X}'_H - R_{eo} \cos AZ \sin \eta \vec{Y}'_H + R_{eo} \cos \eta \vec{Z}_H \quad (25)$$

where components of  $\vec{R}_{eo}$  in horizontal reference frame are

$$\begin{aligned} R_{XH} &= R_{eo} \sin AZ \sin \eta \\ R_{YH} &= -R_{eo} \cos AZ \sin \eta \\ R_{ZH} &= R_{eo} \cos \eta \end{aligned} \quad (26)$$

By substituting and collecting terms, the equation for  $\vec{R}$  is

$$\vec{R} = (R_{xH} + X) \vec{X}'_H + (R_{yH} + Y) \vec{Y}'_H + (R_{zH} + Z) \vec{Z}'_H \quad (27)$$

The vectors  $\vec{R}$  and  $\vec{\omega}_E$  have now been expressed in the horizontal reference frame. The development of equation (15) can proceed further by taking the first derivative of  $R$ , with respect to time, in the following general form for an inertial coordinate system:

$$\left( \frac{d\vec{R}}{dt} \right) = \left[ \frac{d\vec{R}}{dt} \right] + \vec{\omega} \times \vec{R}$$

where

$\left[ \frac{d\vec{R}}{dt} \right]$  is the derivative relative to the rotating reference system

$\left( \frac{d\vec{R}}{dt} \right)$  is the vehicle CG velocity vector with respect to the inertial reference frame (R)

Therefore,

$$\begin{aligned} \frac{d\vec{R}}{dt} &= (\dot{R}_{xH} + \dot{X}) \vec{X}'_H + (\dot{R}_{yH} + \dot{Y}) \vec{Y}'_H + (\dot{R}_{zH} + \dot{Z}) \vec{Z}'_H \\ &+ \vec{\omega}_E \times \left[ (\dot{R}_{xH} + \dot{X}) \vec{X}'_H + (\dot{R}_{yH} + \dot{Y}) \vec{Y}'_H + (\dot{R}_{zH} + \dot{Z}) \vec{Z}'_H \right] \quad (28) \end{aligned}$$

where a dot above the variable denotes first derivative with respect to time.

The last term may be expanded in the following matrix form:

$$\begin{aligned} \vec{\omega}_E \times \vec{R} &= \begin{vmatrix} \vec{X}'_H & \vec{Y}'_H & \vec{Z}'_H \\ \omega_{ExH} & \omega_{EyH} & \omega_{EzH} \\ (R_{xH} + X) & (R_{yH} + Y) & (R_{zH} + Z) \end{vmatrix} \\ &= \begin{bmatrix} \omega_{EyH} (R_{zH} + Z) - \omega_{EzH} (R_{yH} + Y) \\ \omega_{EzH} (R_{xH} + X) - \omega_{ExH} (R_{zH} + Z) \\ \omega_{ExH} (R_{yH} + Y) - \omega_{EyH} (R_{xH} + X) \end{bmatrix} \begin{bmatrix} \vec{X}'_H \\ \vec{Y}'_H \\ \vec{Z}'_H \end{bmatrix} \\ &= \begin{bmatrix} \omega_{EyH} (R_{zH} + Z) - \omega_{EzH} (R_{yH} + Y) \\ \omega_{EzH} (R_{xH} + X) - \omega_{ExH} (R_{zH} + Z) \\ \omega_{ExH} (R_{yH} + Y) - \omega_{EyH} (R_{xH} + X) \end{bmatrix} \begin{bmatrix} \vec{X}'_H \\ \vec{Y}'_H \\ \vec{Z}'_H \end{bmatrix} \end{aligned}$$

From this expanded form, the following equations can be established:

$$A = \omega_{E_{yH}} (R_{zH} + Z) - \omega_{E_{zH}} (R_{yH} + Y) \quad (29a)$$

$$B = \omega_{E_{zH}} (R_{xH} + X) - \omega_{E_{xH}} (R_{zH} + Z) \quad (29b)$$

$$C = \omega_{E_{xH}} (R_{yH} + Y) - \omega_{E_{yH}} (R_{xH} + X) \quad (29c)$$

Therefore, the velocity vector is

$$\frac{d\vec{R}}{dt} = (\dot{X} + A) \vec{X}'_H + (\dot{Y} + B) \vec{Y}'_H + (\dot{Z} + C) \vec{Z}_H \quad (30)$$

To obtain the acceleration, the second derivative of  $\vec{R}$  with respect to time is taken

$$\begin{aligned} \frac{d^2\vec{R}}{dt^2} = & (\ddot{X} + \dot{A}) \vec{X}'_H + (\ddot{Y} + \dot{B}) \vec{Y}'_H + (\ddot{Z} + \dot{C}) \vec{Z}_H \\ & + \omega_E \times \left[ (\dot{X} + A) \vec{X}'_H + (\dot{Y} + B) \vec{Y}'_H + (\dot{Z} + C) \vec{Z}_H \right] \end{aligned} \quad (31)$$

where  $\dot{A}$ ,  $\dot{B}$ ,  $\dot{C}$  are the derivatives of equations (29a), (29b), and (29c),

$$\dot{A} = \omega_{E_{yH}} \dot{Z} - \omega_{E_{zH}} \dot{Y} \quad (32a)$$

$$\dot{B} = \omega_{E_{zH}} \dot{X} - \omega_{E_{xH}} \dot{Z} \quad (32b)$$

$$\dot{C} = \omega_{E_{xH}} \dot{Y} - \omega_{E_{yH}} \dot{X} \quad (32c)$$

and the last term is expanded to give

$$\begin{aligned}
\vec{\omega}_E \times \frac{d\vec{R}}{dt} &= \begin{vmatrix} \vec{X}'_H & \vec{Y}'_H & \vec{Z}'_H \\ \omega_{E_{xH}} & \omega_{E_{yH}} & \omega_{E_{zH}} \\ (\dot{X} + A) & (\dot{Y} + B) & (\dot{Z} + C) \end{vmatrix} \\
&= \left[ \omega_{E_{yH}} (\dot{Z} + C) - \omega_{E_{zH}} (\dot{Y} + B) \right] \vec{X}'_H \\
&\quad + \left[ \omega_{E_{zH}} (\dot{X} + A) - \omega_{E_{xH}} (\dot{Z} + C) \right] \vec{Y}'_H \\
&\quad + \left[ \omega_{E_{xH}} (\dot{Y} + B) - \omega_{E_{yH}} (\dot{X} + A) \right] \vec{Z}'_H
\end{aligned}$$

By substituting the above into equation (31), the following is obtained for the acceleration

$$\begin{aligned}
\frac{d^2\vec{R}}{dt^2} &= \left[ \ddot{X} + \omega_{E_{yH}} \dot{Z} - \omega_{E_{zH}} \dot{Y} + \omega_{E_{yH}} (\dot{Z} + C) - \omega_{E_{zH}} (\dot{Y} + B) \right] \vec{X}'_H \\
&\quad + \left[ \ddot{Y} + \omega_{E_{zH}} \dot{X} - \omega_{E_{xH}} \dot{Z} + \omega_{E_{zH}} (\dot{X} + A) - \omega_{E_{xH}} (\dot{Z} + C) \right] \vec{Y}'_H \\
&\quad + \left[ \ddot{Z} + \omega_{E_{xH}} \dot{Y} - \omega_{E_{yH}} \dot{X} + \omega_{E_{xH}} (\dot{Y} + B) - \omega_{E_{yH}} (\dot{X} + A) \right] \vec{Z}'_H \quad (33)
\end{aligned}$$

By collecting terms and reducing equation (33), the final development of vehicle acceleration with respect to the horizontal reference frame is

$$\frac{d^2\vec{R}}{dt^2} = \left( \ddot{X} + 2\dot{A} + \omega_{E_{yH}} C - \omega_{E_{yH}} B \right) \vec{X}'_H$$

$$\begin{aligned}
 & + \left( \ddot{Y} + 2\dot{B} + \omega_{EzH} A - \omega_{ExH} C \right) \vec{Y}_H' \\
 & + \left( \ddot{Z} + 2\dot{C} + \omega_{ExH} B - \omega_{EyH} A \right) \vec{Z}_H'
 \end{aligned} \quad (34)$$

or in component form in the horizontal reference frame,

$$a_x = \ddot{X} + 2\dot{A} + \omega_{EyH} C - \omega_{EzH} B \quad (35a)$$

$$a_y = \ddot{Y} + 2\dot{B} + \omega_{EzH} A - \omega_{ExH} C \quad (35b)$$

$$a_z = \ddot{Z} + 2\dot{C} + \omega_{ExH} B - \omega_{EyH} A \quad (35c)$$

This completes the derivation of the acceleration equations, which includes coriolis and centripetal acceleration due to rotation of the coordinate system, and accelerations of the vehicle CG due to forces and moments.

To complete the equations of motion, it will be necessary to introduce the forces and moments acting on the vehicle. These forces and moments result mainly, for this program, from gravitational attraction, and from reaction of the air on the vehicle as a result of its motion. This is not to say other types of forces cannot be involved.

#### c. Earth Position Coordinates

Since the components of the gravitational force are dependent upon the position of the vehicle in the earth coordinate system, the instantaneous longitude and geocentric latitude can be obtained by the following method.

From equation (6) and reference 9, the geocentric latitude is

$$\lambda_c = \arcsin \frac{z_E}{R_{EQ}}$$

and the longitude is

$$\beta = \arctan \frac{Y_E}{X_E}$$

where  $X_E$ ,  $Y_E$ , and  $Z_E$  are coordinates in the earth reference system.

This program expresses the vehicle position in the horizontal reference frame; therefore, the position must be transformed to the earth reference frame. This can be done through examination of figures 2, 5, and 6, and the following transformation matrixes.

The order of transformation will be from  $X'_H$ ,  $Y'_H$ ,  $Z'_H$  to  $X_H$ ,  $Y_H$ , and  $Z_H$ , to  $X_E$ ,  $Y_E$ , and  $Z_E$ .

Therefore,

$$\begin{aligned} X_H &= X'_H \sin AZ - Y'_H \cos AZ \\ Y_H &= X'_H \cos AZ + Y'_H \sin AZ \\ Z_H &= Z'_H \end{aligned} \quad (36a)$$

or in matrix form

$$\begin{bmatrix} X_H \\ Y_H \\ Z_H \end{bmatrix} = \begin{bmatrix} \sin AZ & (-\cos AZ) & 0 \\ \cos AZ & \sin AZ & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X'_H \\ Y'_H \\ Z'_H \end{bmatrix} \quad (36b)$$

and

$$\begin{aligned} X_E &= X_H \sin \lambda_g + Z_H \cos \lambda_g \\ Y_E &= Y_H \\ Z_E &= -X_H \cos \lambda_g + Z_H \sin \lambda_g \end{aligned} \quad (37a)$$

from which the following matrix is formed:

$$\begin{bmatrix} X_e \\ Y_e \\ Z_e \end{bmatrix} = \begin{bmatrix} \sin \lambda_g & 0 & \cos \lambda_g \\ 0 & 1 & 0 \\ -\cos \lambda_g & 0 & \sin \lambda_g \end{bmatrix} \cdot \begin{bmatrix} X_H \\ Y_H \\ Z_H \end{bmatrix} \quad (37b)$$

By substituting equation (36b) into equation (37b)

$$\begin{bmatrix} X_e \\ Y_e \\ Z_e \end{bmatrix} = \begin{bmatrix} \sin \lambda_g & 0 & \cos \lambda_g \\ 0 & 1 & 0 \\ -\cos \lambda_g & 0 & \sin \lambda_g \end{bmatrix} \cdot \begin{bmatrix} \sin AZ & (-\cos AZ) & 0 \\ \cos AZ & \sin AZ & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X'_H \\ Y'_H \\ Z'_H \end{bmatrix} \quad (38)$$

and expanding equation (38),

$$\begin{bmatrix} X_e \\ Y_e \\ Z_e \end{bmatrix} = \begin{bmatrix} (\sin \lambda_g \sin AZ) & (-\sin \lambda_g \cos AZ) & \cos \lambda_g \\ \cos AZ & \sin AZ & 0 \\ (-\cos \lambda_g \sin AZ) & (\cos \lambda_g \cos AZ) & \cos \lambda_g \end{bmatrix} \cdot \begin{bmatrix} X'_H \\ Y'_H \\ Z'_H \end{bmatrix} \quad (39)$$

From equation (39)

$$X_E = X'_H (\sin \lambda_g \sin AZ) - Y'_H (\sin \lambda_g \cos AZ) + Z'_H \cos \lambda_g \quad (40a)$$

$$Y_E = X'_H \cos AZ + Y'_H \sin AZ \quad (40b)$$

$$Z_E = -X'_H (\cos \lambda_g \sin AZ) + Y'_H (\cos \lambda_g \cos AZ) + Z'_H \sin \lambda_g \quad (40c)$$

where

$$X'_H = R_{xH} + X$$

$$Y'_H = R_{yH} + Y$$

$$Z'_H = R_{zH} + Z$$

$R_{xH}$ ,  $R_{yH}$ , and  $R_{zH}$  are defined by equation (26). Therefore, the instantaneous geocentric latitude can be obtained from

$$\lambda_c = \arcsin \left( \frac{Z_E}{R} \right) \quad (41)$$

where

$$R = \left[ (R_{xH} + X)^2 + (R_{yH} + Y)^2 + (R_{zH} + Z)^2 \right]^{1/2}$$

since the earth is assumed to be a body of revolution about the polar axis, the instantaneous longitude may be defined as

$$\beta = \arctan \left( \frac{Y_E}{X_E} \right) \quad (42)$$

and will be considered positive when measured east from the prime meridian and negative when measured west.

#### d. Gravitational Equations

The potential function and equations used to obtain the gravitational forces to be included in the equations of motion are presented in this subsection. The gravitational potential of the earth is an expansion of spherical harmonics as a function of latitude and longitude. However, in this program, the earth is assumed to be a body of revolution so that the longitudinal effects may be ignored. Although some satellite data indicate that the earth's equator is elliptical where the major axis is somewhat larger than the minor axis, the difference though is very small compared to the equatorial radius.

The gravity components are determined by taking partial derivatives of the gravitational potential equation (Ref. 9).

$$\begin{aligned} \phi(R, \lambda) = & \frac{-G M_{00}}{R} \left[ 1 + \frac{J}{3} \left( \frac{R_{EQ}}{R} \right)^2 (1 - 3 \sin^2 \lambda_c) \right. \\ & + \frac{H}{5} \left( \frac{R_{EQ}}{R} \right)^3 (3 \sin \lambda_c - 5 \sin^3 \lambda_c) \\ & \left. + \frac{K}{30} \left( \frac{R_{EQ}}{R} \right)^4 (3 - 30 \sin^2 \lambda_c + 35 \sin^4 \lambda_c) + \dots \right] \end{aligned} \quad (43)$$



where

$G M_o = 1.4076427 \times 10^{16} \text{ ft}^3/\text{sec}^2$  gravitational constant of the earth

$R$  = radial distance to body CG from earth's geocenter

$R_{EQ}$  = earth's equatorial radius

$\lambda_c$  = geocentric latitude

$J = 1623.41 \times 10^{-6}$ , coefficient of the second harmonic

$H = 6.04 \times 10^{-6}$ , coefficient of the third harmonic

$K = 6.37 \times 10^{-6}$ , coefficient of the fourth harmonic

The gravitational potential field is usually defined by spherical harmonics. Each higher order term in equation (43) is due to a deviation of the potential from that of a true sphere. Therefore, the potential is expressed as a series of Legendre polynomials in the variable,  $\lambda_c$ . Many analyses (Refs. 9, 11, 12) consider only the second-, third-, and fourth-order terms. The first harmonic ( $R_{EQ}/K$ ) is missing because of the choice of an equatorial coordinate system in which  $\lambda_c$  is measured relative to the center of the earth. The fifth harmonic is not sufficiently well known to justify its inclusion in this equation.

The gravitational acceleration of a vehicle along any line is the partial derivative of the potential function,  $\phi$ , along that line. See figure 7.

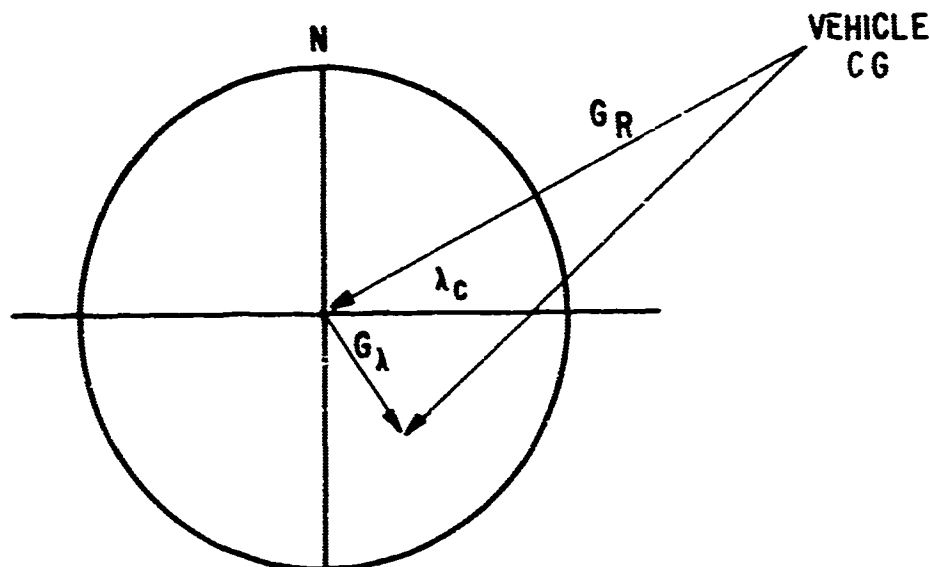


Figure 7. Gravitational Direction

$\vec{g}$  is related to  $\phi(R, \lambda_c)$  by

$$\vec{g} = -\nabla\phi(R, \lambda_c)$$

where  $\nabla\phi$  is the gradient vector of  $\phi(R, \lambda_c)$ . From figure 7, the force along the radius, R, is

$$G_R = mg_R = -m \frac{\partial\phi(R, \lambda_c)}{\partial R}$$

and in  $R, \lambda_c$  direction

$$G_{\lambda} = mg_{\lambda c} = -\frac{m}{R} \frac{\partial\phi(R, \lambda_c)}{\partial \lambda_c}$$

where  $m$  = mass of vehicle in slugs. From equation (43),  $G_R$  and  $G_{\lambda c}$  are obtained.

$$\begin{aligned} G_R &= -m \frac{\partial\phi}{\partial R} = +m G_o M_o \left[ -\frac{1}{R^2} - \frac{3J}{3} \left( \frac{R_{EQ}^2}{R^4} \right) (1 - 3 \sin^2 \lambda_c) \right. \\ &\quad \left. - \frac{4}{5} H \left( \frac{R_{EQ}^3}{R^5} \right) (3 \sin \lambda_c - 5 \sin^3 \lambda_c) \right. \\ &\quad \left. - \frac{5K}{30} \left( \frac{R_{EQ}^4}{R^6} \right) (3 - 30 \sin^2 \lambda_c + 35 \sin^4 \lambda_c) \right] \\ G_{\lambda c} &= -\frac{m}{R} \frac{\partial\phi}{\partial \lambda_c} = \frac{+m G_o M_o}{R^2} \left[ \frac{J}{3} \left( \frac{R_{EQ}^2}{R^2} \right) (-6 \sin \lambda_c \cos \lambda_c) \right. \\ &\quad \left. + \frac{H}{5} \left( \frac{R_{EQ}^3}{R^3} \right) (3 \cos \lambda_c - 15 \sin^2 \lambda_c \cos \lambda_c) \right. \\ &\quad \left. + \frac{K}{30} \left( \frac{R_{EQ}^4}{R^4} \right) (-60 \sin \lambda_c \cos \lambda_c + 140 \sin^3 \lambda_c \cos \lambda_c) \right] \end{aligned}$$

simplifying and dividing through by  $m$ ,

$$G_R = - \frac{G M_o}{R^2} \left[ 1 + J \left( \frac{R_{EQ}}{R} \right)^2 (1 - 3 \sin^2 \lambda_c) \right. \\ \left. + \frac{4H}{5} \left( \frac{R_{EQ}}{R} \right)^3 (3 \sin \lambda_c - 5 \sin^3 \lambda_c) \right. \\ \left. + \frac{K}{6} \left( \frac{R_{EQ}}{R} \right)^4 (3 - 30 \sin^2 \lambda_c + 35 \sin^4 \lambda_c) \right] \quad (44a)$$

$$G\lambda_c = - \frac{G M_o}{R^2} \left[ -2 J \left( \frac{R_{EQ}}{R} \right)^2 (\sin \lambda_c \cos \lambda_c) \right. \\ \left. + \frac{3H}{5} \left( \frac{R_{EQ}}{R} \right)^3 (\cos \lambda_c - 5 \sin^2 \lambda_c \cos \lambda_c) \right. \\ \left. + \frac{2K}{3} \left( \frac{R_{EQ}}{R} \right)^4 (-3 \sin \lambda_c \cos \lambda_c + 7 \sin^3 \lambda_c \cos \lambda_c) \right] \quad (44b)$$

Since  $G_R$  is along the radius  $\vec{R}$  and  $G\lambda_c$  is perpendicular to  $\vec{R}$ , they are in the inertial reference frame. For them to be included in the equations of motion, their components must be found with respect to the horizontal reference frame. This can be accomplished by the use of direction cosines and transformation matrices.

After examination of figure 8, the following series of equations can be written to transpose  $G_R$  and  $G\lambda_c$  to the horizontal reference frame.

It is noted that  $G_R$  and  $G\lambda$  are taken as positive in the direction shown in figures 7 and 8. Therefore, by definition

$$GR = + \frac{\partial \phi}{\partial R} \\ G\lambda = + \frac{1}{R} \frac{\partial \phi}{\partial \lambda} \\ G\beta = 0 \quad (45)$$

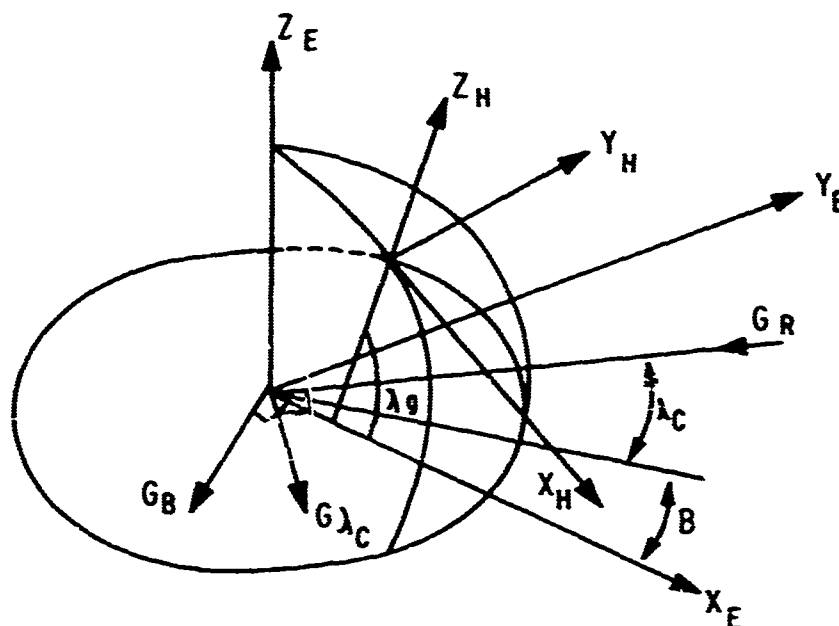


Figure 8. Gravitational Transformations

$$\begin{aligned}GX_E &= -GR \left( \cos \lambda_c \cos \delta \right) + GL \left( \sin \lambda_c \cos \delta \right) + G\delta \sin \delta \\GY_E &= -GR \left( \cos \lambda_c \sin \delta \right) + GL \left( \sin \lambda_c \sin \delta \right) - G\delta \cos \delta \\GZ_E &= -GR \sin \lambda_c - GL \cos \lambda_c\end{aligned}\tag{46}$$

$$\begin{aligned}GX_H &= GX_E \sin \lambda_g + GZ_E \cos \lambda_g \\GY_H &= GY_E \\GZ_H &= GX_E \cos \lambda_g + GZ_E \sin \lambda_g\end{aligned}\tag{47}$$

From equations (16a), (16b), and (16c), the following is obtained:

$$\begin{aligned}GX'_H &= GX_H \sin AZ + GY_H \cos AZ \\GY'_H &= -GX_H \cos AZ + GY_H \sin AZ \\GZ'_H &= GZ_H\end{aligned}\tag{48}$$

Equations (46) and (47) can be expressed in matrix form

$$\begin{bmatrix} GX_H \\ GY_H \\ GZ_H \end{bmatrix} = \begin{bmatrix} (-\sin \lambda_g \cos \lambda_c \cos \beta + \cos \lambda_g \sin \lambda_c) \\ (-\cos \lambda_c \sin \beta) \\ (-\cos \lambda_g \cos \lambda_c \cos \beta - \sin \lambda_g \sin \lambda_c) \end{bmatrix}$$

$$\begin{bmatrix} (\sin \lambda_g \sin \lambda_c \cos \beta + \cos \lambda_g \cos \lambda_c) & (\sin \lambda_g \sin \beta) \\ (\sin \lambda_c \sin \beta) & (-\cos \beta) \\ (\cos \lambda_g \sin \lambda_c \cos \beta - \sin \lambda_g \cos \lambda_c) & (\cos \lambda_g \sin \beta) \end{bmatrix} \cdot \begin{bmatrix} GR \\ G\lambda \\ GB \end{bmatrix} \quad (49)$$

This matrix may be written into the following equations for transformation:

$$\begin{aligned} G11 &= -\sin \lambda_g \cos \lambda_c \cos \beta + \cos \lambda_g \sin \lambda_c \\ G12 &= \sin \lambda_g \sin \lambda_c \cos \beta + \cos \lambda_g \cos \lambda_c \\ G21 &= -\cos \lambda_c \sin \beta \\ G22 &= \sin \lambda_c \sin \beta \\ G31 &= -\cos \lambda_g \cos \lambda_c \cos \beta - \sin \lambda_g \sin \lambda_c \\ G32 &= +\cos \lambda_g \sin \lambda_c \cos \beta - \sin \lambda_g \cos \lambda_c \end{aligned} \quad (50)$$

The final transformation is obtained by substituting equation (49) into equation (48).

$$\begin{aligned} GX_H' &= GR (G11 \cdot \sin AZ + G21 \cdot \cos AZ) \\ &\quad + G\lambda (G12 \cdot \sin AZ + G22 \cdot \cos AZ) \\ GY_H' &= GR (-G11 \cdot \cos AZ + G21 \cdot \sin AZ) \\ &\quad + G\lambda (-G12 \cdot \cos AZ + G22 \cdot \sin AZ) \\ GZ_H' &= GZ_H = GR (G31) + G\lambda (G32) \end{aligned} \quad (51)$$

The components of the gravitational forces are now expressed in the horizontal reference frame and may be included with equations (35) to complete the equations of motion.

Before the aerodynamic forces and moments can be resolved, a method must be established for the transformation of components from the body reference frame to the horizontal reference frame and the inverse.

#### e. Euler Angles

Up to this point, only the equations for the three-translational degrees-of-freedom have been derived. This establishes the vehicle CG with respect to a horizontal reference system. To analyze the body forces and moments as a result of its motion about the CG, the angular orientation of the vehicle must be known with respect to the horizontal reference system.

The Euler angles describing the angular motion are  $\phi$ ,  $\theta$ , and  $\psi$ , and will be noted as pitch, yaw, and roll angles, respectively. These angles are to be differentiated from the angles of attack and sideslip. The angles of attack and sideslip have a kinematic definition based on the components of the free-stream velocity relative to the body axes of the vehicle. The angular displacements,  $\phi$ ,  $\theta$ , and  $\psi$ , on the other hand, are used to measure the vehicle attitude with respect to a fixed set of axes (X, Y, and Z), and in no way require motion of the vehicle relative to the surrounding air for their definition.

Consider a vehicle moving with respect to the inertial axes X, Y, and Z which are also fixed in space. The following will describe one of several ways of specifying the angular orientation of a vehicle at any instant of time. This will be done by successively pitching, yawing, and rolling the X, Y, and Z axes until they coincide with the axes x, y, and z of the vehicle as shown in figure 9.

First, pitch the vehicle by an angular displacement  $\phi$  around OX so that Z goes to  $Z_1$  and Y goes to  $Y_1$  (figure 9a). The relationship between the two coordinate systems is then given by the following transfer matrix:

$$\begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi & \cos \phi \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad (52)$$

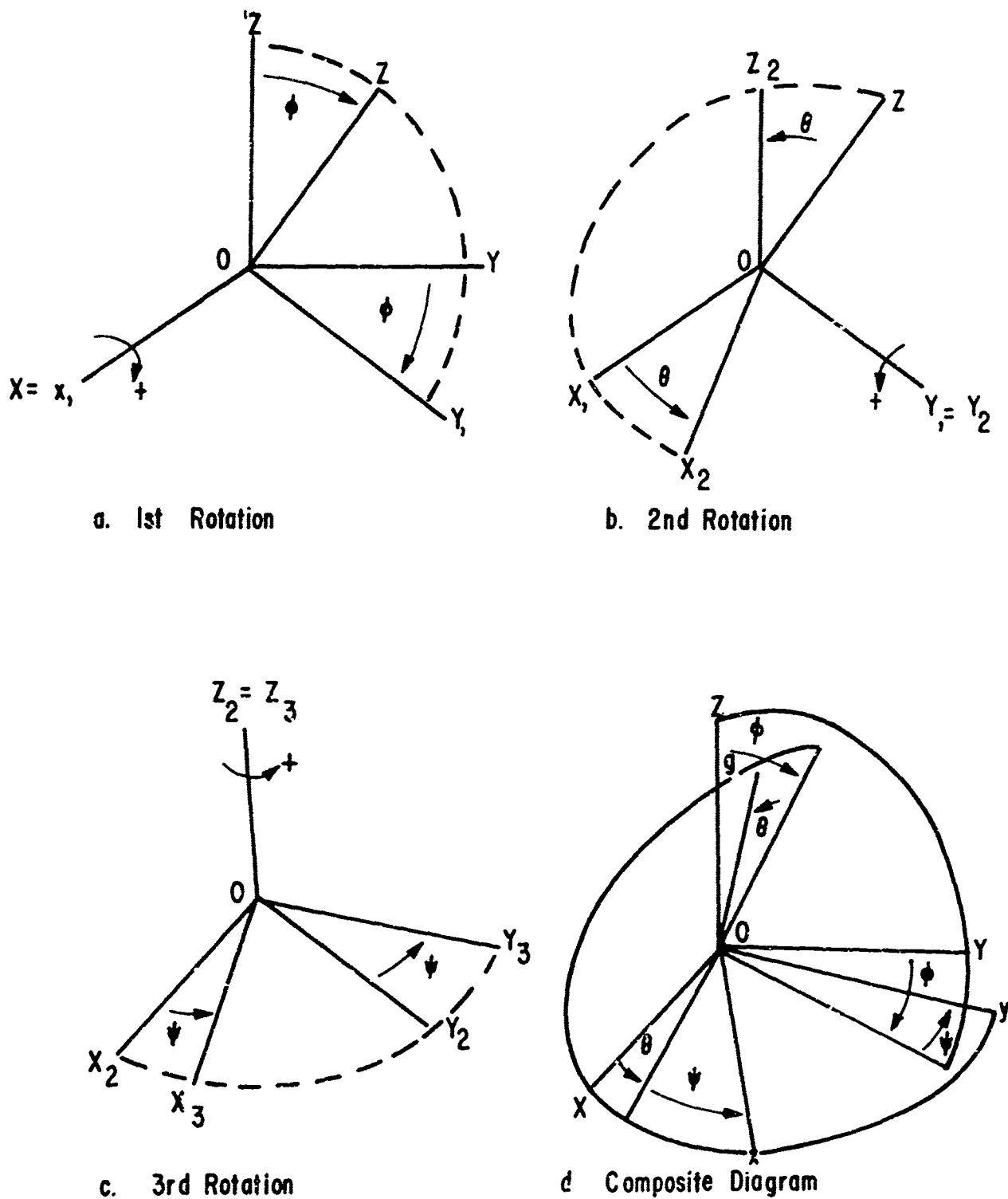


Figure 9. System of Angular Displacements

Next, yaw the vehicle by an angle  $\theta$  about the  $OY_1$  axes so that  $X_1$  goes to  $X_2$  and  $Z_1$  to  $Z_2$  (figure 9b). The new relationship is given by the transfer matrix

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} \quad (53)$$

Finally, roll the vehicle by angle  $\psi$  around the  $OZ_2$  axis so that  $X_2$  moves to  $X_3$ , and  $Y_2$  to  $Y_3$  (figure 9c). The transfer matrix for this rotation is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} \quad (54)$$

where

$$x = X_3$$

$$y = Y_3$$

$$z = Z_3$$

The operations of pitch, yaw, and roll for this program must always be performed in this order since angular displacements do not follow the ordinary laws of vector addition, but, in fact, follow a noncommutative law. Under this system of rotations, the direction cosines of the final vehicle axes  $x$ ,  $y$ , and  $z$  to the fixed axis  $X$ ,  $Y$ , and  $Z$  are obtained by substituting equation (52) into equations (53).

$$\begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} = \begin{bmatrix} \cos \theta & (-\sin \theta \sin \psi) & (-\sin \theta \cos \psi) \\ 0 & \cos \psi & -\sin \psi \\ \sin \theta & (\sin \psi \cos \theta) & (\cos \psi \cos \theta) \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad (55)$$



Then substitute equation (54) into equation (55)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (\cos \psi \cos \theta) & (-\sin \theta \sin \phi \cos \psi + \sin \psi \cos \theta) \\ (-\sin \psi \cos \theta) & (\sin \theta \sin \phi \sin \psi + \cos \psi \cos \phi) \\ \sin \theta & (\cos \theta \sin \phi) \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad (56)$$

Therefore, equation (56) will transform components from the fixed axes or horizontal reference system to the vehicle or body reference system. To transform components from the vehicle axes to the horizontal reference axes, the matrix of equation (56) must be inverted by interchanging rows and columns. The elements of equation (56) may be written for simplicity as follows, where subscripts indicate rows and columns.

$$\begin{aligned} A_{11} &= \cos \theta \cos \psi \\ A_{12} &= -\sin \theta \sin \phi \cos \psi + \sin \psi \cos \theta \\ A_{13} &= -\sin \theta \cos \phi \cos \psi - \sin \psi \sin \phi \\ A_{21} &= -\sin \psi \cos \theta \\ A_{22} &= \sin \theta \sin \phi \sin \psi + \cos \psi \cos \phi \\ A_{23} &= \sin \psi \sin \theta \cos \phi - \sin \phi \cos \psi \\ A_{31} &= \sin \theta \\ A_{32} &= \cos \theta \sin \phi \\ A_{33} &= \cos \theta \cos \phi \end{aligned} \quad (57)$$

#### f. Transformation of Angular Velocities

It is now necessary to express the angular velocities  $\omega_x$ ,  $\omega_y$ , and  $\omega_z$  with respect to the vehicle axes in terms of the Euler angles.

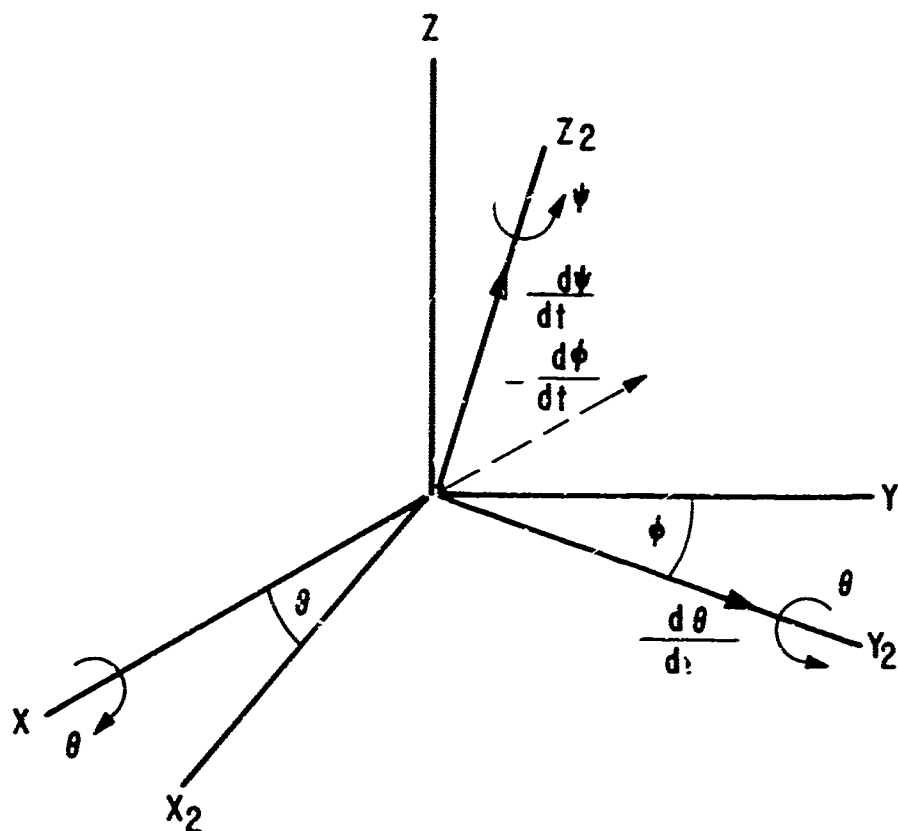


Figure 10. Angular Velocities of Euler Angles

The components of the angular velocities along the  $X_2$ ,  $Y_2$ , and  $Z_2$  axes to the vehicle axes  $x$ ,  $y$ , and  $z$  shown in figure 10 are

$$\begin{aligned}\omega_x &= \frac{-d\phi}{dt} \cos \theta \cos \psi + \frac{d\psi}{dt} \sin \psi \\ \omega_y &= \frac{d\phi}{dt} \cos \theta \sin \psi + \frac{d\psi}{dt} \cos \psi \\ \omega_z &= \frac{-d\theta}{dt} \sin \theta + \frac{d\psi}{dt}\end{aligned}\quad (58)$$

The equations in section II-2e and equation (56) have been derived with respect to the inertial axes  $X$ ,  $Y$ , and  $Z$ . If the axes system is allowed to rotate, the angular velocity of the earth  $\omega_E$  must be included. Therefore, the inertial angular rates of the vehicle are

$$\begin{aligned}\Omega_x &= \omega_x + \omega_{E_{XH}} \\ \Omega_y &= \omega_y + \omega_{E_{YH}} \\ \Omega_z &= \omega_z + \omega_{E_{ZH}}\end{aligned}\quad (59)$$

where  $\omega_{E_{XH}}$ ,  $\omega_{E_{YH}}$ , and  $\omega_{E_{ZH}}$  are defined by equation (19). These are the components of the earth's rotation rate in the horizontal reference frame.

Since this program was written for a symmetrical cone with uniform weight distribution and further assumes symmetrical ablation, then the products of inertia will be zero regardless of roll angle,  $\psi$ . (This will be shown in section II-2g.) Therefore, the roll and transverse moment of inertia can be obtained independent of  $\psi$ . This will simplify the equations and allow the angular momentum,  $\vec{h}_O$ , to be written in the second rotation.

Therefore, equation (58) can be simplified and written in the  $X_2$ ,  $Y_2$ , and  $Z_2$  axes as

$$\begin{aligned}\omega_{x_2} &= -\frac{d\phi}{dt} \cos \theta \\ \omega_{y_2} &= \frac{d\theta}{dt} \\ \omega_{z_2} &= -\frac{d\phi}{dt} \sin \theta + \frac{d\psi}{dt}\end{aligned}\quad (60)$$

which are the total rotation rates of the vehicle in the second rotated axes.

Now  $\omega_{E_{XH}}$ ,  $\omega_{E_{YH}}$ , and  $\omega_{E_{ZH}}$  must also be written in the same axes system as the vehicle angular rates. This can be done by using the transformation equation (55), which will result in

$$\begin{aligned}\omega_{E_{x_2}} &= \omega_{E_{xH}} \cos \theta - \omega_{E_{yH}} \sin \theta \sin \phi - \omega_{E_{zH}} \sin \theta \cos \phi \\ \omega_{E_{y_2}} &= 0 + \omega_{E_{yH}} \cos \phi - \omega_{E_{zH}} \sin \phi \\ \omega_{E_{z_2}} &= \omega_{E_{xH}} \sin \theta + \omega_{E_{yH}} \sin \phi \cos \theta + \omega_{E_{zH}} \cos \phi \cos \theta\end{aligned}\quad (61)$$

Therefore, equation (59) can be written as

$$\begin{aligned}\Omega_x &= \omega_{x_2} + \omega_{Ex_2} \\ \Omega_y &= \omega_{y_2} + \omega_{Ey_2} \\ \Omega_z &= \omega_{z_2} + \omega_{Ez_2}\end{aligned}\quad (62)$$

which are the inertial angular velocities of the vehicle in the  $X_2$ ,  $Y_2$ , and  $Z_2$  axes.

Since equation (62) is written in the second rotated axes system, it is also necessary to have the angular velocity of that coordinate system. Therefore,

$$\begin{aligned}\Omega_{x_2} &= -\frac{d\theta}{dt} \cos \theta + \omega_{Ex_2} \\ \Omega_{y_2} &= \frac{d\theta}{dt} + \omega_{Ey_2} \\ \Omega_{z_2} &= -\frac{d\theta}{dt} \sin \theta + \omega_{Ez_2}\end{aligned}\quad (63)$$

which completes the transformation of angular velocities.

#### g. Moment of Momentum

A particle of mass,  $m$ , moving with velocity,  $\vec{\dot{R}}$ , figure 11, has a linear momentum (Ref. 13)

$$\vec{P} = m \vec{\dot{R}}$$

The moment of this linear momentum about an arbitrary point,  $O$ , is defined as

$$\vec{h}_O = \vec{r} \times m \vec{\dot{R}} \quad (64)$$

where

$\vec{\dot{R}}$  = absolute velocity of  $m$

$\vec{r}$  = radius drawn from  $O$  as shown in figure 11

$\vec{h}_O$  = the angular momentum of the particle about the point  $O$

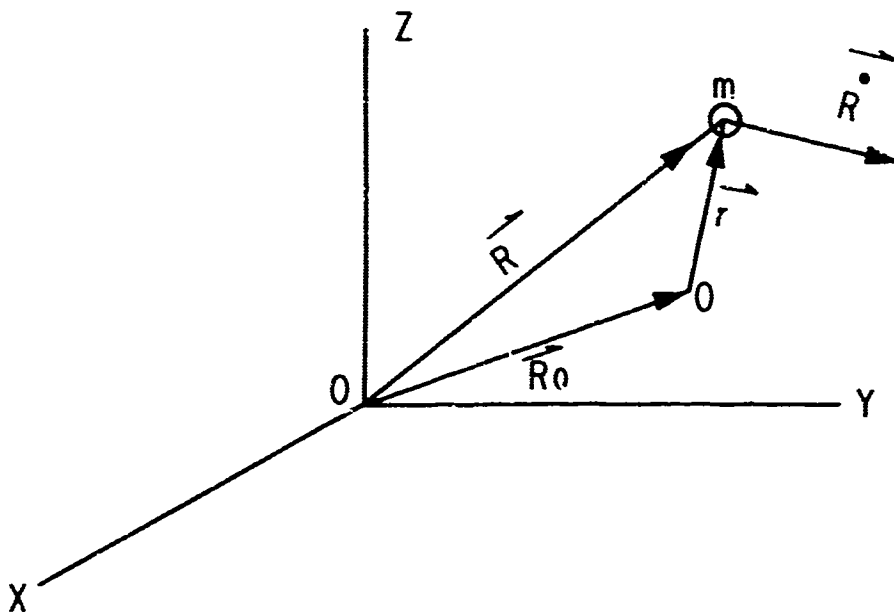


Figure 11. Moment of Momentum about 0

The angular momentum of a rigid body, figure 12, about any axis perpendicular to the plane of motion and passing through the point 0 fixed in a moving body is the sum of the moments of the linear momenta of all its particles about the axis. Therefore, equation (64) can be rewritten as

$$\vec{h}_0 = \sum_i \vec{r}_i \times m \vec{v}_i$$

where  $\vec{v}_i$  = velocity of any point  $i$  on the body. The velocity of a representative particle of mass  $m_i$  may be expressed in terms of the velocity of 0 plus the velocity of  $m_i$  with respect to 0.

$$\vec{v}_i = \vec{v}_0 + \vec{\omega} \times \vec{r}_i$$

Therefore,

$$\vec{h}_0 = \sum_i \vec{r}_i \times m_i (\vec{v}_0 + \vec{\omega} \times \vec{r}_i)$$

or is expanded from

$$\vec{h}_0 = \sum_i \vec{r}_i \times (\vec{\omega} \times \vec{r}_i) m_i - \vec{v}_0 \times \sum_i m_i \vec{r}_i$$

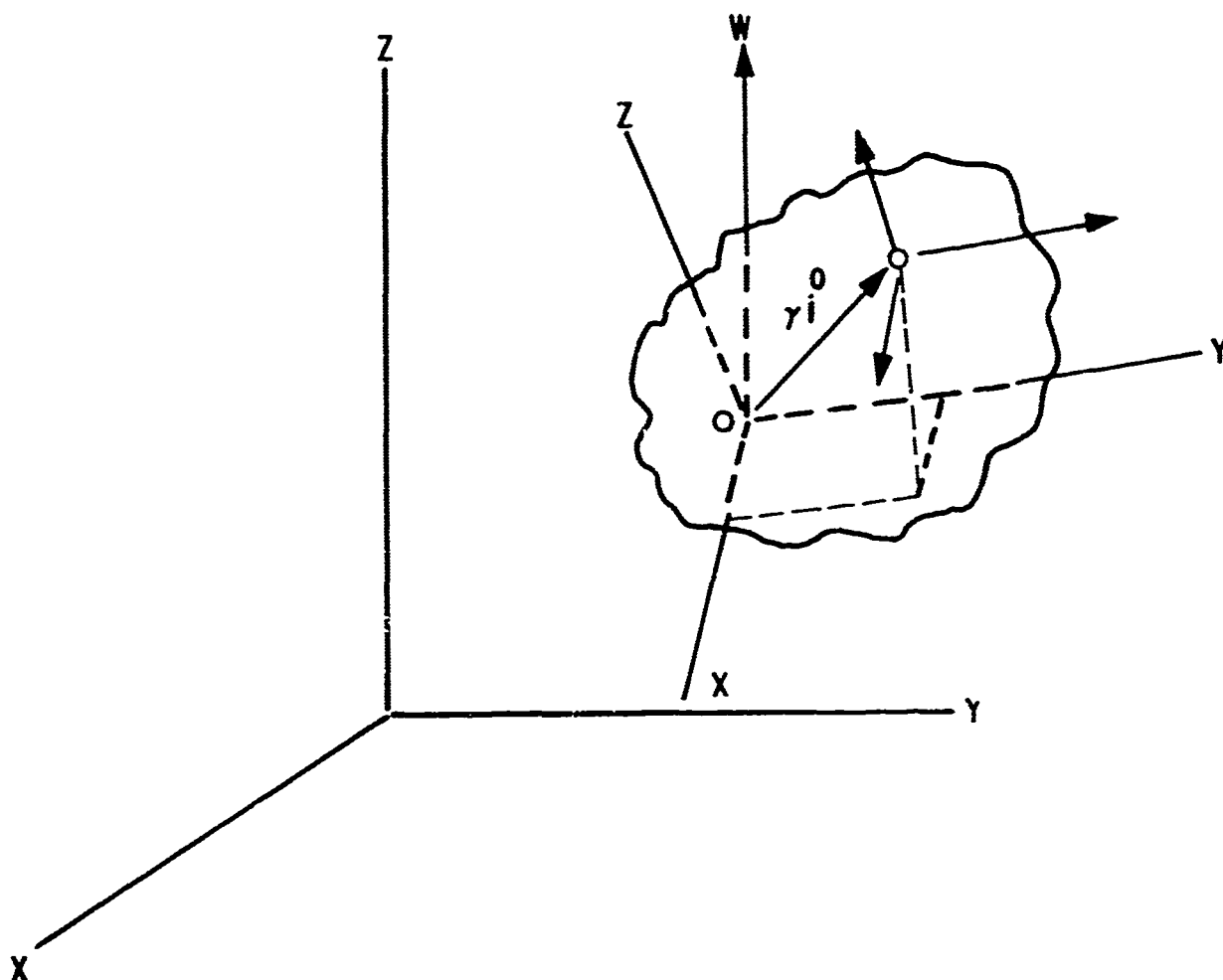


Figure 12. Components of Momentum

If, by definition,  $O$  coincides with the center of mass, the term  $\sum_i m_i \dot{\mathbf{r}}_i$  is equal to zero, the angular momentum can be expressed by the following integral

$$\vec{h}_O = \int \vec{r} \times (\vec{\omega} \times \vec{r}) \, dm \quad (65)$$

The relation between the angular momentum of a body and the applied moments,  $\vec{M}_O$ , of the forces  $\sum \vec{F} = m\vec{R}$  acting on  $m$  is obtained from rotational equation of motion.

$$\vec{M}_O = \sum \vec{r} \times m\vec{R} \quad (66)$$

Taking the second derivative of the expression

$$\vec{R} = \vec{R}_0 + \vec{r}$$

and substituting into equation (66)

$$\vec{M}_0 = \sum \vec{r} \times m \left( \ddot{\vec{R}}_0 + \ddot{\vec{r}} \right)$$

or

$$\vec{M}_0 = \sum \frac{d}{dt} (\vec{r} \times m \dot{\vec{r}}) - \ddot{\vec{R}}_0 \times \sum m \vec{r} \quad (67)$$

Again  $\sum m \vec{r} = 0$  when 0 coincides with center of mass. Equation (67) reduces to

$$\vec{M}_0 = \sum \frac{d}{dt} (\vec{r} \times m \dot{\vec{r}}) \quad (68)$$

but  $\dot{\vec{r}}$  is equal to the following:

$$\dot{\vec{r}} = \left( \frac{d\vec{r}}{dt} \right) = \left[ \frac{d\vec{r}}{dt} \right] + \vec{\omega} \times \vec{r}$$

where

$$\left( \frac{d\vec{r}}{dt} \right) = \text{total derivative relative to inertial axis}$$

$$\left[ \frac{d\vec{r}}{dt} \right] = \text{derivative relative to rotating axis and, since body is rigid, is equal to zero}$$

Therefore,

$$\dot{\vec{r}} = \vec{\omega} \times \vec{r}$$

and substituting into equation (67) the relationship between  $\vec{M}_0$  and  $\vec{h}_0$  is established.

$$\vec{M}_0 = \frac{d}{dt} \left( \vec{r} \times m (\vec{\omega} \times \vec{r}) \right) \quad (69)$$

or

$$\vec{M}_O = \frac{d \vec{h}_O}{dt} \quad (70)$$

With this relationship established, the derivation of  $\vec{h}_O$  can be continued. The integral in equation (65) must be evaluated and this is done by first expanding the cross product  $\vec{\omega} \times \vec{r}$

$$\vec{\omega} \times \vec{r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \Omega_x & \Omega_y & \Omega_z \\ x & y & z \end{vmatrix} = (\Omega_y z - \Omega_z y) \vec{i} + (\Omega_z x - \Omega_x z) \vec{j} + (\Omega_x y - \Omega_y x) \vec{k} \quad (71)$$

where  $\Omega_x$ ,  $\Omega_y$ , and  $\Omega_z$  is defined by equation (62). Then expansion of the cross product  $\vec{r} \times (\vec{\omega} \times \vec{r})$  is

$$\begin{aligned} \vec{r} \times (\vec{\omega} \times \vec{r}) &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ (\Omega_y z - \Omega_z y) & (\Omega_z x - \Omega_x z) & (\Omega_x y - \Omega_y x) \end{vmatrix} \\ &= \vec{i} \left[ \Omega_x (y^2 + z^2) - \Omega_y (xy) - \Omega_z (xz) \right] \text{cm} \\ &+ \vec{j} \left[ -\Omega_x (xy) + \Omega_y (x^2 + z^2) - \Omega_z (yz) \right] \text{cm} \\ &+ \vec{k} \left[ -\Omega_x (xz) + \Omega_y (yz) + \Omega_z (x^2 + y^2) \right] \text{cm} \quad (72) \end{aligned}$$

By definition, the moments of inertia of a body about the x, y, z axes are

$$\begin{aligned} I_x &= \int (y^2 + z^2) \text{cm} \\ I_y &= \int (x^2 + z^2) \text{cm} \\ I_z &= \int (x^2 + y^2) \text{cm} \end{aligned}$$



and the products of inertia are

$$I_{xy} = \int xy \, dm$$

$$I_{xz} = \int xz \, dm$$

$$I_{yz} = \int yz \, dm$$

If the x, y, z axes are considered the principal axes, the products of inertia and the time derivatives of the moments of inertia are zero.

Therefore, by integrating equation (72) over the entire body and applying the above definition, the following is obtained:

$$\vec{h}_o = I_x \Omega_x \vec{i} + I_y \Omega_y \vec{j} + I_z \Omega_z \vec{k} \quad (73)$$

in which case the moment of momentum components along the x, y, z axes are

$$h_x = I_x \Omega_x$$

$$h_y = I_y \Omega_y$$

$$h_z = I_z \Omega_z \quad (74)$$

It was shown previously that the moment about a body's center of gravity was equal to the time derivative of the moment of momentum.

$$\vec{M}_o = \frac{d \vec{h}_o}{dt}$$

which in general form can be expressed as

$$\left( \frac{d \vec{h}_o}{dt} \right) = \left[ \frac{d \vec{h}_o}{dt} \right] + \vec{\omega} \times \vec{h}_o$$

Substitute this into equation (70), Euler's moment equation is obtained,

$$\vec{M}_o = \frac{d \vec{h}_o}{dt} + \vec{\Omega}_2 \times \vec{h}_o \quad (75)$$

where  $\vec{\Omega}_2$  is the angular velocity of the second rotated axes system.

Taking the cross product  $\vec{\Omega}_2 \times \vec{h}_0$  in a manner similar to equation (71), equation (75) becomes

$$\begin{aligned} \vec{M}_0 = & \left[ \frac{dhx}{dt} \vec{i} + \frac{dhy}{dt} \vec{j} + \frac{dhz}{dt} \vec{k} \right] + (\Omega_{y2} h_z - \Omega_{z2} h_y) \vec{i} \\ & + (\Omega_{z2} h_x - \Omega_{x2} h_z) \vec{j} + (\Omega_{x2} h_y - \Omega_{y2} h_x) \vec{k} \end{aligned} \quad (76)$$

and collecting terms yields

$$\begin{aligned} \vec{M}_0 = & \left( \frac{dhx}{dt} + \Omega_{y2} h_z - \Omega_{y2} h_y \right) \vec{i} + \left( \frac{dhy}{dt} + \Omega_{z2} h_x - \Omega_{x2} h_z \right) \vec{j} \\ & + \left( \frac{dhz}{dt} + \Omega_{x2} h_y - \Omega_{y2} h_x \right) \vec{k} \end{aligned} \quad (77)$$

Now, taking the derivatives of equation (74) and remembering  $d(I)/dt = 0$  for principal axes,

$$\begin{aligned} \frac{dhx}{dt} &= I_x \dot{\Omega}_x \\ \frac{dhy}{dt} &= I_y \dot{\Omega}_y \\ \frac{dhz}{dt} &= I_z \dot{\Omega}_z \end{aligned} \quad (78)$$

where  $\dot{\Omega}_x$ ,  $\dot{\Omega}_y$ ,  $\dot{\Omega}_z$  are the derivatives of equation (62)

$$\begin{aligned} \dot{\Omega}_x &= -\ddot{\phi} \cos \theta + \dot{\phi} \dot{\theta} \sin \theta + \dot{\omega}_{Ex_2} \\ \dot{\Omega}_y &= \ddot{\phi} + \dot{\omega}_{Ey_2} \\ \dot{\Omega}_z &= -\ddot{\phi} \sin \theta - \dot{\phi} \dot{\theta} \cos \theta + \dot{\omega}_{Ez_2} \end{aligned} \quad (79)$$

and  $\dot{\omega}_{Ex_2}$ ,  $\dot{\omega}_{Ey_2}$ ,  $\dot{\omega}_{Ez_2}$  are the derivatives of equation (61). Before taking the derivatives of equation (61), let the following be elements of equation (55) where subscripts indicate row and columns:

$$\begin{aligned}
B_{11} &= \cos \theta \\
B_{12} &= -\sin \theta \sin \phi \\
B_{13} &= -\sin \theta \cos \phi \\
B_{21} &= 0 \\
B_{22} &= \cos \phi \\
B_{23} &= -\sin \phi \\
B_{31} &= \sin \theta \\
B_{32} &= \cos \theta \sin \phi \\
B_{33} &= \cos \theta \cos \phi
\end{aligned} \tag{80}$$

The derivatives of equation (80) are defined here

$$\begin{aligned}
DB_{11} &= -\dot{\theta} \sin \theta \\
DB_{12} &= -\dot{\theta} \cos \theta \sin \phi - \dot{\phi} \sin \theta \cos \phi \\
DB_{13} &= -\dot{\theta} \cos \theta \cos \phi + \dot{\phi} \sin \theta \sin \phi \\
DB_{21} &= 0 \\
DB_{22} &= -\dot{\phi} \sin \phi \\
DB_{23} &= -\dot{\phi} \cos \phi \\
DB_{31} &= \dot{\theta} \cos \theta \\
DB_{32} &= -\dot{\theta} \sin \theta \sin \phi + \dot{\phi} \cos \theta \cos \phi \\
DB_{33} &= -\dot{\theta} \sin \theta \cos \phi - \dot{\phi} \cos \theta \sin \phi
\end{aligned} \tag{81}$$

Therefore,

$$\begin{aligned}
\dot{\omega}_{Ex_2} &= DB_{11}\dot{\omega}_{Ex} + DB_{12}\dot{\omega}_{Ey} + DB_{13}\dot{\omega}_{Ez} \\
\dot{\omega}_{Ey_2} &= DB_{21}\dot{\omega}_{Ex} + DB_{22}\dot{\omega}_{Ey} + DB_{23}\dot{\omega}_{Ez} \\
\dot{\omega}_{Ez_2} &= DB_{31}\dot{\omega}_{Ex} + DB_{32}\dot{\omega}_{Ey} + DB_{33}\dot{\omega}_{Ez}
\end{aligned} \tag{82}$$

Substituting equations (74), (75), and (79) into equation (77) and writing  $\vec{M}_0$  in component form with  $I_x = I_y$ ,

$$\begin{aligned}
 M_x &= I_x \left( -\ddot{\phi} \cos \theta + \dot{\phi} \dot{\theta} \sin \theta + \dot{\omega}_{Ex_2} \right) + \Omega_{y2} (I_z \Omega_z) - \Omega_{z2} (I_x \Omega_y) \\
 M_y &= I_x \left( \ddot{\theta} + \dot{\omega}_{Ey_2} \right) + \Omega_{z2} (I_x \Omega_x) - \Omega_{x2} (I_z \Omega_z) \\
 M_z &= I_z \left( -\ddot{\phi} \sin \theta - \dot{\phi} \dot{\theta} \cos \theta + \ddot{\psi} + \dot{\omega}_{Ez_2} \right) + \Omega_{x2} (I_x \Omega_y) \\
 &\quad - \Omega_{y2} (I_x \Omega_x)
 \end{aligned} \tag{83}$$

#### h. Summary of Equations of Motion

For convenience, the equations of motion are restated here with all terms added and in a form similar to that used in the program.

$$\frac{d^2x}{dt^2} = \frac{Fx}{m} - 2\dot{A} - \omega_{EyH} C + \omega_{EzH} B - Gx$$

$$\frac{d^2y}{dt^2} = \frac{Fy}{m} - 2\dot{B} - \omega_{EzH} A + \omega_{ExH} C - Gy$$

$$\frac{d^2z}{dt^2} = \frac{Fz}{m} - 2\dot{C} - \omega_{ExH} B + \omega_{EyH} A - Gz$$

$$\frac{d^2\theta}{dt^2} = \frac{My}{I_x} - \Omega_{z2} \Omega_x + \frac{\Omega_{x2} I_z \Omega_z}{I_x} - \dot{\omega}_{Ey_2}$$

$$\frac{d^2\phi}{dt^2} = \left( \frac{-\dot{\lambda}x}{I_x} + \frac{\Omega_{y2} I_z \Omega_z}{I_x} - \Omega_{z2} \Omega_y + \frac{d\dot{\phi}}{dt} \frac{d\theta}{dt} \sin \theta + \dot{\omega}_{Ex_2} \right) / \cos \theta$$

$$\frac{d^2\psi}{dt^2} = \frac{Mz}{I_z} - \frac{\Omega_{x2} I_x \Omega_y}{I_z} + \frac{\Omega_{y2} I_x \Omega_x}{I_z} + \frac{d^2\dot{\psi}}{dt^2} \sin \theta$$

$$+ \frac{d\dot{\phi}}{dt} \frac{d\theta}{dt} \cos \theta - \dot{\omega}_{Ez_2}$$

where all term are known from the initialization or from the previous time step.

Although these equations have been written for a conical shaped body with weight change due only to ablation, it only a matter of modifying the equations to include thrust and additional forces and moments associated with a finned missile.

### 3. INTEGRATION TECHNIQUE

To integrate a set of  $n$  simultaneous second order differential equations of the form

$$\frac{d^2 x_i}{dt^2} = f_i(t, x_1, x_2, \dots, x_n), \quad i = (1, 2, \dots, n)$$

The Runge-Kutta method of numerical integration is recommended for this type of application and especially where a high degree of accuracy is desired. Thus, the equations of motion in the preceding subsection are shown as six second-order differential equations, which are to be solved simultaneously. This method consists of four sets of equations which involve different substitutions into the differential equations. The equations are solved, and the increments of the functions are calculated as weighted averages of the solutions to the four equations. Therefore, the integration procedure for the expression of the form

$$\frac{d^2 x}{dt^2} = f(t, x, x')$$

is described by the following generalized equations

$$x_{n+1} = x_n + x'_n$$

$$x'_{n+1} = x'_n + \frac{\Delta t}{6} (k_0 + 2k_1 + 2k_3 + k_3)$$

where

$$k_0 = \Delta t f(t_n, x_n, x'_n)$$

$$k_1 = \Delta t f\left(t_n + \frac{\Delta t}{2}, x_n + \frac{\Delta t}{2} x'_n, x'_n + \frac{k_0}{2}\right)$$

$$k_2 = \Delta t f \left( t_n + \frac{\Delta t}{2}, x_n + \left( x'_n + \frac{\Delta t}{2} k \right) \frac{\Delta t}{2}, x'_n + \frac{k_1}{2} \right)$$

$$k_3 = \Delta t f \left( t_n + \Delta t, x_n + x'_n \Delta t + \frac{\Delta t}{2} k_1, x'_n + k_2 \right)$$

In integrating a differential equation numerically, the equation is replaced by a difference equation and solved accordingly. The Runge-Kutta method has excellent properties for stability in the integration and if the integration step is taken small enough, the difference equation is usually close to the differential equation solution. However, some variables in the equations of motion may have peculiar oscillations or increase very rapidly which could cause an unstable condition or an error in the solution. An unstable condition or solution can often be made stable by using a smaller integration step. The reason for this is that different step sizes change the parameters relating to the stability of the difference solution.

The integration step size,  $\Delta t$ , for this program is fixed, although the user could modify the integration method to use a variable step size under error control (Ref. 14). The user has the option to break into the program at exact specified values of the independent variable,  $t$ , for the printout step.

#### 4. AERODYNAMIC HEATING AND ABLATION

The equations of heat transfer are general in that they apply to most axially symmetric vehicles of any composite skin structure. The vehicle skin is assumed to be made of discrete layers of material whose properties may vary from layer to layer and there is no limit on the number of layers. This program is specific in that the equations have been applied to a blunt-nosed, conical vehicle.

Hypersonic vehicles may be categorized into two general types. The first consists of those vehicles which must be propelled through the atmosphere to sustain flight. The second type comprises those that have a vast store of kinetic and potential energy which will be dissipated to the surrounding air during reentry. The latter type is of primary interest and for which this program was written, although it will handle either type.

In either case, some of this energy is expended against drag forces which assume the form of skin friction and pressure drag which produce heat. The heat equivalent of the kinetic energy contained within a body is approximately

$$V_{en}^2 \quad 50,000 \text{ (Btu/lb)}$$

where  $V_{en}$  is the reentry velocity in ft/sec. While only a fraction of this energy is actually transferred to the body as heat, this fraction is still a significant quantity and accurately illustrates the magnitude of the problem.

a. Heat Flux Equation

Experiments in high speed flow have verified that the magnitude and direction of heat flow at the surface does not depend on the difference between the wall temperature and the free-stream temperature as in low-speed flow, but rather on the difference between the wall temperature,  $T_W$ , and the adiabatic wall temperature,  $T_{AW}$ . It is apparent that the determination of the adiabatic wall temperature will be of prime importance in the calculation of heat transfer, since the transference of heat to or from the wall will depend upon whether the skin temperature is above or below  $T_{AW}$ . The adiabatic wall temperature can conveniently be expressed in terms of a dynamic-temperature rise

$$T_{AW} = T_{EMPE} + RF \left( \frac{V_E^2}{2g_c C_{pa} J} \right) \quad (84)$$

where

$T_{EMPE}$  = boundary layer (B/L) edge temperature, °R

$V_E$  = B/L edge velocity, ft/sec

$g_c$  = gravitational constant, ft/sec<sup>2</sup>

$J$  = Joule's constant = 778

$C_{pa}$  = specific heat of air, Btu/lb-°R

$RF$  = recovery factor which is a measure of the fraction of the free-stream dynamic temperature rise recovered at the wall

Reference 15 shows that for practical purposes

$$RF \approx \sqrt{PR} \quad (85a)$$

for laminar flow and

$$RF \approx \sqrt[3]{PR} \quad (85b)$$

for turbulent flow where

PR\* = Prandtl number

$$PR = C_{pa} \mu^*/K^*$$

$\mu^*$  = viscosity of air in lb/ft-sec evaluated at reference temperature ( $T_{REF}$ )

$K^*$  = thermal conductivity of air in Btu/ft-sec-°R evaluated at  $T_{REF}$

Therefore, the unit surface convective heat rate for high-speed flow (Ref. 16) is

$$\dot{q}_c/A = \bar{h}_e (T_{AW} - T_w) \quad (86)$$

where

$\dot{q}_c/A$  = heating rate in Btu/sec-ft<sup>2</sup>

$\bar{h}_e$  = convective heat transfer coefficient in Btu/ft<sup>2</sup>-sec-°R

A = local wetted area in ft<sup>2</sup>

#### (1) Heat Transfer Coefficient

For determining the heat transfer coefficient in laminar flow over a flat plate, an empirical method using Blasius theory was developed by Rubesin and Johnson (Ref. 17) to account for the combined effects of the local Mach number and the temperature ratio of the wall,  $T_w$ , to the boundary layer edge, TEMPE. In this method, the conventional analysis of heat transfer by convection can be used, but the reference temperature,  $T_{REF}$ , for evaluating the fluid properties is expressed as a function of the local Mach number and of the ratio  $T_w/TEMPE$ . The method has been extended to cover turbulent flow, and has proved to be both convenient and useful.

Therefore, for laminar flow the reference temperature is

$$T_{REF} = TEMPE \left[ 1 + 0.58 \left( \frac{T_w}{TEMPE} - 1 \right) + 0.032 \text{ MACHE}^2 \right] \quad (87)$$

and for turbulent flow

$$T_{REF} = TEMPE \left[ 1 + 0.45 \left( \frac{T_w}{TEMPE} - 1 \right) + 0.035 \text{ MACHE} \right] \quad (88)$$



Equations (87) and (88) can be expressed in the form for laminar flow as

$$T_{REF} = TEMPE + 0.58 (T_W - TEMPE) + 0.19 (T_{AW} - TEMPE) \quad (89)$$

and for turbulent flow as

$$T_{REF} = TEMPE + 0.45 (T_W - TEMPE) + 0.21 (T_{AW} - TEMPE) \quad (90)$$

The heat transfer coefficient (Refs. 16, 18) is related to the flow properties through the expression

$$Nu^*(PR^*)^{1/3} = C(R_e)^a \quad (91)$$

where

$N_u^*$  = Nusselt number =  $h_e X / K^*$

$h_e$  = effective heat transfer coefficient

$X$  = distance from nose, ft

$K^*$  = thermal conductivity of air, Btu/ft-sec-°R, evaluated at  $T_{REF}$

$R_e$  = B/L edge Reynolds number =  $\rho_e V_e X / \mu_e$

$\rho_e$  = boundary layer edge density, lb-sec<sup>2</sup>/ft<sup>4</sup>

$V_e$  = boundary layer edge velocity, ft/sec

$C = 0.575$  for laminar flow on cone

$C = 0.0296$  for turbulent flow on cone

$C = 0.778$  for laminar flow on blunt body

$C = 0.0348$  for turbulent flow on blunt body

$a = 0.8$  for turbulent flow

$a = 0.5$  for laminar flow

$\mu_e$  = B/L edge viscosity, lb-sec/ft<sup>2</sup>

The solutions to these equations for the heat transfer coefficient is obtained by the use of the Blasius incompressible flat plate skin friction coefficients modified for compressible flow by use of Eckert's reference enthalpy. For

laminar flow

$$\frac{C_{fc}}{C_f} = \sqrt{\frac{\rho^* u^*}{\rho_e u_e}} \quad (92a)$$

and for turbulent flow

$$\frac{C_{fc}}{C_f} = \left( \frac{u^*}{u_e} \right)^{0.2} \left( \frac{\rho^*}{\rho_e} \right)^{0.8} \quad (92b)$$

Good correlation with experimental data is obtained if the gas properties are evaluated at the reference temperature and the velocity is taken as the boundary layer edge velocity. For the case of mass addition, the properties of the injected gas are used to compute the Prandtl number and Nusselt number.

It is important in the selection of a suitable material for thermal protection by the ablative process that the products of decomposition have a high specific heat. This in turn produces a high effective mean specific heat of the gas-air mixture in the boundary layer and a high Prandtl number. It is also desirable that the ablator have a low thermal conductivity. This will decrease heat conducted to the interior of the vehicle. The surface of the vehicle also receives heat by radiation from the hot gas in the shock layer in addition to that by convection. As the air density is low, its emissivity is much less than 1 percent. Therefore, the rate of radiation heat transfer is negligible compared with that of aerodynamic convection and can be ignored.

#### b. Unsteady Heat Conduction

A means of applying the Schmidt graphical method for solving an unsteady heat conduction problem is presented here. A thorough discussion of this method can be found in any general text (Refs. 16, 19) and will not be attempted here. This method is quite flexible in that difficult boundary conditions can be handled easily.

This method is widely used for solving unsteady heat conduction problems because it gives an iterative profile of the temperature change. Because of its simplicity, mistakes, if they occur, are easily found, and relatively untrained personnel can accomplish the work. Numerical methods are especially convenient when a high-speed digital computer is available since the steps in a numerical solution can be programmed relatively easily.

The numerical method for solving unsteady-state heat conduction differs from that used to solve steady state. In the latter case, the temperature distribution in a body can be obtained for a network of points in a solid by solving a system of residual equations. In unsteady-state systems, the initial temperature profile is known, but its variation with time must be determined. Therefore, it is necessary to resolve the temperature profile at some future time from a given distribution at an earlier time.

To illustrate the numerical method, it is necessary to transform the Fourier conduction equation (a partial differential equation that is second order in space and first order in time) for the unsteady temperature distribution in a heat-conducting solid into a finite difference form.

$$\frac{\Delta T}{\Delta \tau} = \frac{\alpha \Delta^2 T}{(\Delta x)^2} \quad (93)$$

The subscripts denote the differentiation variable. Letting  $n$  denote position and  $t$  time,  $\Delta T$  can be written as

$$\Delta T = T_{n,t+1} - T_{n,t}$$

and in a similar manner

$$\Delta^2 T = T_{n+1,t} - 2T_{n,t} + T_{n-1,t}$$

The expression  $\Delta^2 T$  thus becomes

$$\Delta^2 T = T_{n+1,t} - 2T_{n,t} + T_{n-1,t}$$

Substituting these expressions into the Fourier equation (93), gives

$$T_{n,t+1} - T_{n,t} = \frac{\alpha \Delta \tau}{(\Delta x)^2} (T_{n+1,t} - 2T_{n,t} + T_{n-1,t}) \quad (94)$$

The temperature throughout a wall or slab can now be computed for any later time if the initial distribution is known. A Schmidt plot demonstrates the temperature profile versus time in a semi-infinite slab (figure 13).

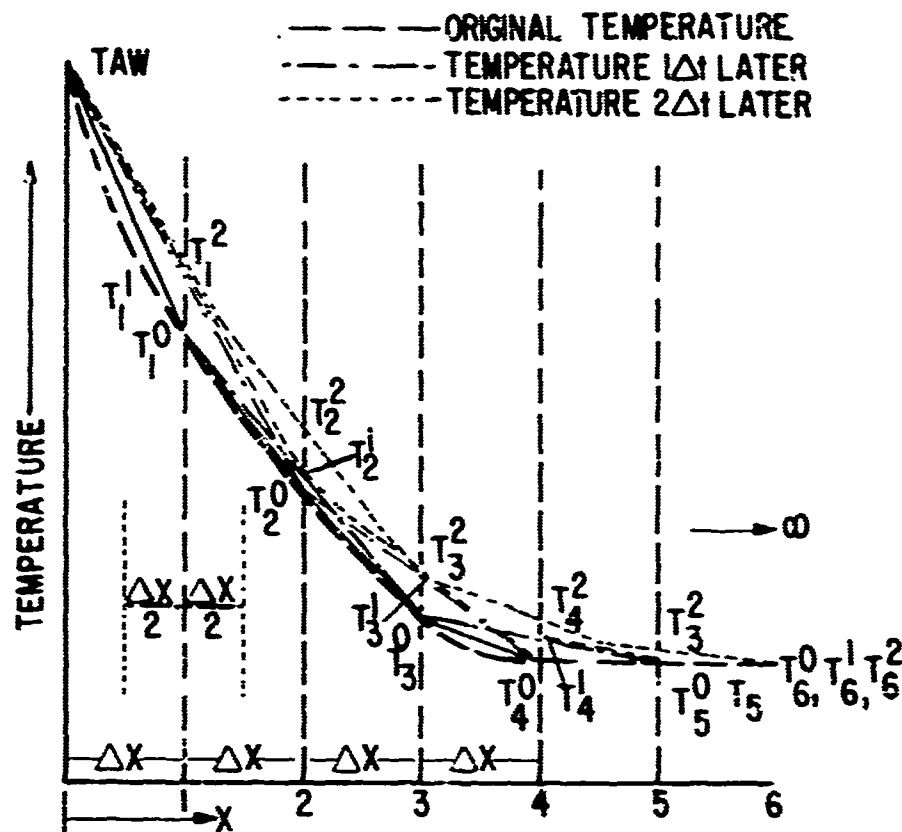


Figure 13. Schmidt Plot in an Infinitely Thick Wall

A constant wall temperature is assumed in the graphical example, but a varying wall temperature can be handled with equal ease. By letting the wall temperature vary with time, the distribution at each point within the body can be computed for each time increment.

#### c. Vehicle Thermal Model

To determine the time-temperature profile in the vehicle skin, the Fourier conduction equation was written as a finite difference equation and solved numerically. This method is a further adaptation of the Schmidt graphical method (Ref. 19). The wall is divided into a number of lamina with known thickness, specific heat, and thermal conductivity and with a known initial temperature distribution. Figure 14 illustrates the model with its associated nomenclature.

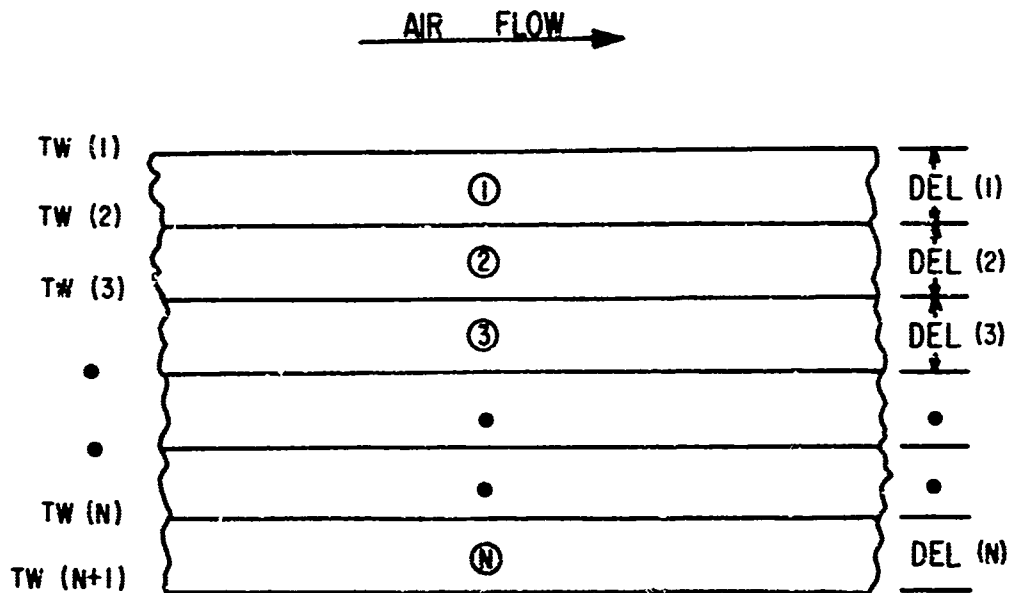


Figure 14. Model for Wall Temperature Profile and Ablation Recession Calculations

The difference equation for computing the temperature at position  $n$  at time  $t + 1$  is

$$T_{n,t+1} = T_{n,t} + \frac{\alpha \Delta t}{(\Delta x)^2} (T_{n+1,t} - 2T_{n,t} + T_{n-1,t}) \quad (95)$$

In this form, the material properties cannot vary from layer to layer. If the properties vary from lamina to lamina, the above equation must be written in the form

$$T_{n,t+1} = T_{n,t} + \Delta t \left[ \frac{\alpha_{n-1}}{(\Delta x_{n-1})^2} (T_{n-1,t} - T_{n,t}) - \frac{\alpha_n}{(\Delta x_n)^2} (T_{n,t} - T_{n+1,t}) \right] \quad (96)$$

Where  $\alpha$  is the thermal diffusivity and may be replaced by its defined equivalent,

$$\alpha = \frac{K}{\rho_{cp}} \quad (97)$$

where

$K$  = thermal conductivity of material in Btu/ft-sec-°R

$\rho$  = density of material in lbs/ft<sup>3</sup>

$cp$  = specific heat of material in Btu/lb-°R

This step allows one to easily recognize certain groups of terms as the heat conducted through each lamina and permits the convective heat flux to be used in solving for the wall temperature. The equation for the wall temperature is

$$T_{n-1,t+1} = T_{n-1,t} + \frac{2\Delta t}{(\rho_{cp} \Delta X)_{n-1}} \left[ h_e (T_{AW} - T_{n-1,t}) \right] - \frac{2\Delta t K_{n-1}}{(\rho_{cp} \Delta X^2)_{n-1}} (T_{n-1,t} - T_{n,t}) \quad (98)$$

The backface boundary condition can be specified in several ways. If internal heating (or cooling) is present, the backface may be held constant or allowed to vary in a specified manner. A conservative assumption is that no heat flows through the last lamina. This assumption will cause somewhat more ablation and higher temperatures than would be encountered with a cooled backface or other heat sink material.

#### d. Ablation

Advances in hypersonic atmospheric flight have resulted in environments of extremely high temperatures. Boundary layer temperatures in excess of 10,000°F are characteristic of vehicles entering the atmosphere at hypersonic speeds.

In hypersonic flight, it becomes obvious that capacitance or mass heat-sink protection, though simple, is an inefficient means of contending with the extremely high heat fluxes associated with certain reentries or with sustained

periods of thermal flight. Consequently, other means of cooling or protecting for operating beyond heat-sink limitations must be used. One convenient means of environmental protection and for which this program was primarily written is the ablation cooling method.

The ablative process is illustrated in figure 15. The ablation process works in the following manner: (1) the material or ablator acts as a heat sink; (2) when the critical or melting temperature of the ablator is reached, a thin layer of the material at the surface will begin successively to melt, vaporize, depolymerize, or decompose chemically; and (3) as the material vaporizes, the gaseous products of decomposition enter into the boundary layer. Being relatively cool, as compared to the boundary layer air, the injected gas forms a thin, but effective film that reduces the heat transfer to the vehicle skin. It is important in the selection of a suitable material for thermal protection by the ablative process that the thermal conductivity of the material should be as low as possible so as to confine the high-temperature zone at the surface to the thinnest layer possible. Likewise, to reduce the rate of mass loss, and hence, the required weight of protective ablator, the surface temperature at which decomposition and melting begins should be as high as practicable. The products of decomposition should preferably have a high Prandtl number, since it is defined as ratio of heat storage to heat conduction of a gas. This means it is desirable to have the mean effective specific heat of the gas-air mixture in the boundary layer as large as possible.

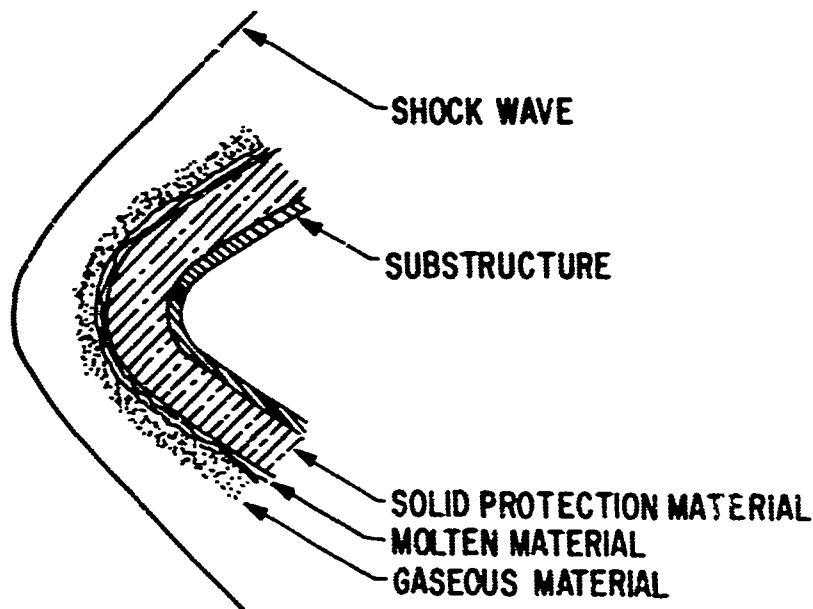


Figure 15. Ablative Process

Since the ablation process can be characterized by an exchange of material for thermal energy, the energy balance at the ablating surface, in its simplest form, is

$$\dot{q}_{\text{net}} = \dot{q}_c - \dot{q}_{\text{cond}} - \dot{q}_{\text{rad}} - \dot{q}_{\text{block}} \quad (99)$$

where  $\dot{q}_c$  is heat transferred to the surface by convection,  $\dot{q}_{\text{cond}}$  is the heat conducted from the surface to the substrate material,  $\dot{q}_{\text{block}}$  is the heat blockage by transpiration in the boundary layer, and  $\dot{q}_{\text{rad}}$  is negligible and has been ignored.

Ablation is assumed to occur when the temperature of the outermost lamella has reached the melting temperature of the material. The outer surface will recede and is assumed to take place normal to the local surface.

#### (1) Mass Loss and Surface Recession

Two mechanisms are involved in the loss of ablation material: (1) mass loss due to oxidation, and (2) mass loss due to sublimation. Oxidation is controlled by the diffusion of oxygen to the reacting carbon, and sublimation is controlled by local pressure and temperature. A detailed discussion on the theory of ablation and material decomposition and reaction is not the intent of this report, so only the results will be presented.

The efficiency of an ablation material is frequently defined for engineering use in terms of a quantity known as the effective heat of ablation,  $Q^*$ , in Btu/lb,

$$Q^* = \frac{\dot{q}_o}{\dot{m}} \quad (100)$$

where  $\dot{q}_o$  is the surface heating rate of a nonablating calorimeter at the ablative temperature, and  $\dot{m}$  is the mass ablation rate. The term "effective heat of ablation" collectively expresses the ability of a material to absorb, block, and dissipate incident heat per unit mass expended. The effective heat of ablation value may also be expressed by using equations (99) and (100) as

$$Q^* = \frac{\dot{q}_{\text{cond}} + \dot{q}_{\text{block}}}{\left(1 - \dot{q}_{\text{rad}}/\dot{q}_o\right)} \quad (101)$$



or by substituting approximate equations for each gives

$$Q^* = \frac{C_p(T_w - T_b) + 0.7 \left( \frac{M_{air}}{M_v} \right)^{1/4} \dot{m}_v (\Delta h)_o}{(1 - \dot{q}_{rad}/\dot{q}_o)} \quad (102)$$

where

$C_p$  = heat capacity of material

$T_w$  = ablative surface temperature

$T_b$  = temperature of unheated material

$M_{air}$  = molecular weight of air

$M_v$  = molecular weight of injected gas

$\dot{m}_v$  = rate of mass injection

$(\Delta h)_o$  = enthalpy difference across boundary layer without transpiration

If the radiation,  $\dot{q}_{rad}$ , is insignificant, equation (102) can be reduced to the form

$$Q^* = A + B (\Delta h)_o \quad (103)$$

where A is an empirical value obtained from experimental investigations of different ablators and B is dependent on whether flow is laminar or turbulent.

The surface recession rate is obtained by

$$\dot{s} = \frac{\dot{q}_c}{Q^* \rho} \quad (104)$$

where

$\dot{s}$  = recession rate, ft/sec

$\rho$  = ablation material density, lb/ft<sup>3</sup>

The enthalpy difference across the boundary can be calculated from the following equations:

$$(\Delta h)_o = h_r - C_{p_2} T_w \quad (105)$$

where

$$h_r = C_{p_a} \cdot T_{EMPE} + R_F \cdot V_E^2 / (2g_c J)$$

The following are analytical expressions used in this program for determining mass loss and recession rates of three common ablators.

(a) Phenolic Refrasil (Silica)

The expression for obtaining the effective heat of ablation for Phenolic Refrasil is

$$Q^* = 4710 + B (\Delta h)_o \quad (106)$$

where

$B = 0.58$  for laminar flow

$B = 0.32$  for turbulent flow

(b) ATJ Graphite

The effective heat of ablation for ATJ graphite is

$$Q^* = 2000 + B (\Delta h)_o \quad (107)$$

where

$B = 2$  for either turbulent or laminar flow

(c) Carbon Phenolic

The expressions for carbon phenolic are more complicated than those used for the other materials due to reaction of carbon with atmospheric oxygen. This material has a very high melting temperature and a low thermal conductivity which gives it excellent insulation properties. Also, its products of decomposition have a high specific heat compared to that of air.

For laminar flow, the diffusion mass loss rate is given by (Refs. 20, 21)

$$\dot{M}_d = \frac{\dot{q}_c}{K1_L + K2_L (h_r - C_{p_{BL}} T_w)} \quad (108)$$

and for turbulent flow, the expression is

$$\dot{M}_d = \frac{\dot{q}_c}{K1_T + K2_T (h_r - C_{p_{BL}} T_W)} \quad (109)$$

where the constants K1 and K2 can be interpreted as the intercept and slope, respectively, on an effective heat of ablation versus enthalpy difference (between recovery and wall conditions) plot.

$\dot{q}_c$  = convective heat flux, Btu/ft<sup>2</sup>-sec

$h_r$  = recovered enthalpy, Btu/lb

$C_{p_{BL}}$  = specific heat in boundary layer, Btu/lb-°R

$T_W$  = wall temperature, -°R

$K1_L$  = 5370

$K2_L$  = 5.37

$K1_T$  = 4240

$K2_T$  = 5.77

The total mass loss rate is given by

$$\dot{M}_T = \dot{M}_d \left[ 1 + 2.64 \times 10^4 (PBL)^{-0.67} \exp \left( \frac{-11.05 \times 10^4}{T_W} \right) \right] \quad (110)$$

where

$\dot{M}_T$  = total mass loss rate, lb/ft<sup>2</sup>-sec

PBL = boundary layer edge pressure, lb/ft<sup>2</sup> (Ref. 20)

The surface recession rate is calculated by

$$\dot{S} = \dot{M}_T / \rho \quad (111)$$

where

$\dot{S}$  = recession rate, ft/sec

$\rho$  = ablation material density, lb/ft<sup>3</sup>

## (2) Total Weight Loss Rate and Total Weight Loss

After the local instantaneous and total local recession rates have been computed, the local weight loss rate can be calculated, if needed, by determining the volume reduction of the local segment and multiplying by the material density. But the primary weight loss of interest is the total for the vehicle. To compute the total vehicle mass loss and mass loss rate, the following procedure is used. Using the local weight loss rate, the instantaneous and total recessions, and the initial radii and slant length of the particular segment of interest (see figure 16), the mass loss rate per unit area is multiplied by the appropriate surface area to give a local integrated mass loss rate. This is accomplished by the following equations:

$$\dot{W}_{T(LEN)} = \pi \left( R_{L(LEN)}^2 + R_{S(LEN)}^2 - 2\dot{C}_{(LEN)} \right) \dot{S}_c SOL_{(LEN)} \quad (112)$$

where

$\dot{W}_{T(LEN)}$  = instantaneous mass loss rate of segment (LEN), lb/sec

(LEN) = number of particular segment of cone

$R_L$  = larger segment radius, ft

$R_S$  = smaller segment radius, ft

$\dot{C}_{(LEN)}$  = total recession of segment, ft

$\dot{S}$  = instantaneous recession of segment, ft/sec

$\rho_c$  = surface material density, lb/ft<sup>3</sup>

$SOL_{(LEN)}$  = slant height of conical segment, ft

$$\dot{W} = \dot{W} + \dot{W}_{T(LEN)} \quad (113)$$

will give total vehicle mass loss rate when the operations in equation (113) are accomplished in a program "Do Loop." Summing these local rates will result in the total vehicle weight loss.

## a. Stagnation Heating

To evaluate stagnation point convective heat transfer rates, this program employs Lee's theory (Ref. 22), modified by use of Eckert's reference enthalpy techniques (Ref. 19), in Lee's equation

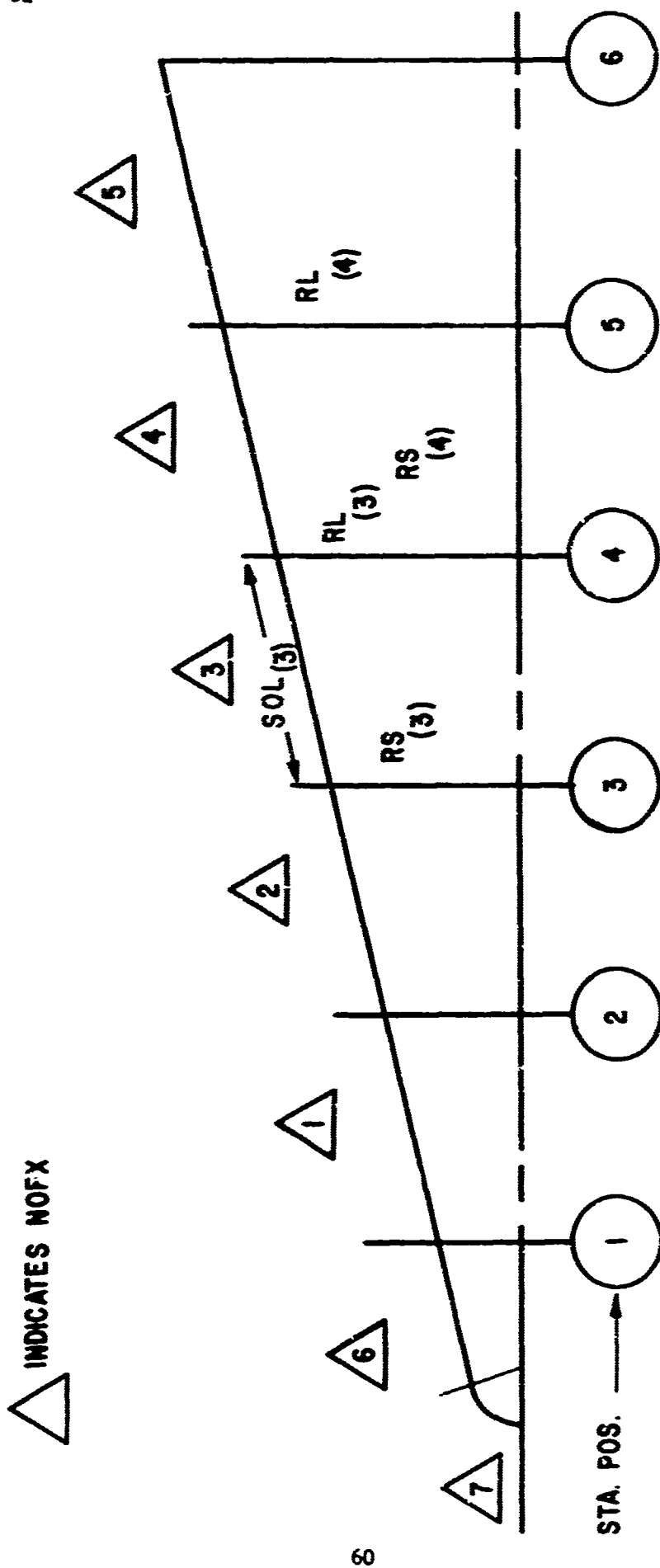


Figure 16. Vehicle Nomenclature

$$q_{c_{STAG}} = \frac{0.778}{(\overline{\gamma})^{2/3}} \sqrt{\rho_{WS} \mu_{WS}} \sqrt{dV/dx} h_{se} \cdot G(M_\infty, \overline{\gamma}, \gamma_\infty) \quad (114)$$

where

$$G(M_\infty, \overline{\gamma}, \gamma_\infty) = \left( \frac{\overline{\gamma}-1}{\overline{\gamma}} \right)^{1/4} \left( 1 + \frac{2}{\gamma_\infty-1} \frac{1}{M_\infty^2} \right)^{1/4} \left( 1 - \frac{1}{\gamma_\infty M_\infty^2} \right)^{1/4}$$

$\overline{\gamma} \approx 1.10 - 1.20$  at high temperatures

$$h_{se} = V_\infty^2 / 2GJ$$

$\gamma_\infty = 1.4$  ratio of specific heats for air

$\rho_{WS}$  = density evaluated at stagnation reference conditions

$\mu_{WS}$  = viscosity evaluated at stagnation reference conditions

$dV/dx$  = velocity component gradient at the stagnation point  
(from Fay and Riddell)

$$\frac{dV}{dx} = \frac{1}{SAR} \left[ 2(P_S - P_\infty) / \rho_{WS} \right]^{0.5}$$

$P_S$  = stagnation pressure (oblique shock)

#### (1) Nose Blunting

In the determination of nose blunting, it is assumed that the initial nose shape of particular interest was a sphere-cone, and after ablation the final shape was a sphere-cone. Experimental evidence has shown that the final eroded shape could be approximated by a sphere-cone having a small deviation from a sphere. The reference sphere-cone configuration after erosion is obtained by placing a spherical surface tangent to a cone parallel to the original surface but displaced by the erosion on the cone and is located axially at a position on the vehicle center line in accordance with the erosion at the stagnation point (figure 17).

The experimental evidence of nose-shape change of reentry vehicles, as previously noted, can be represented by a sphere-cone which deviates from a spherical shape by an amount less than 10 percent of the final nose radius. The final eroded nose radius can be obtained in terms of the stagnation point erosion and cone erosion as shown in the following equation:

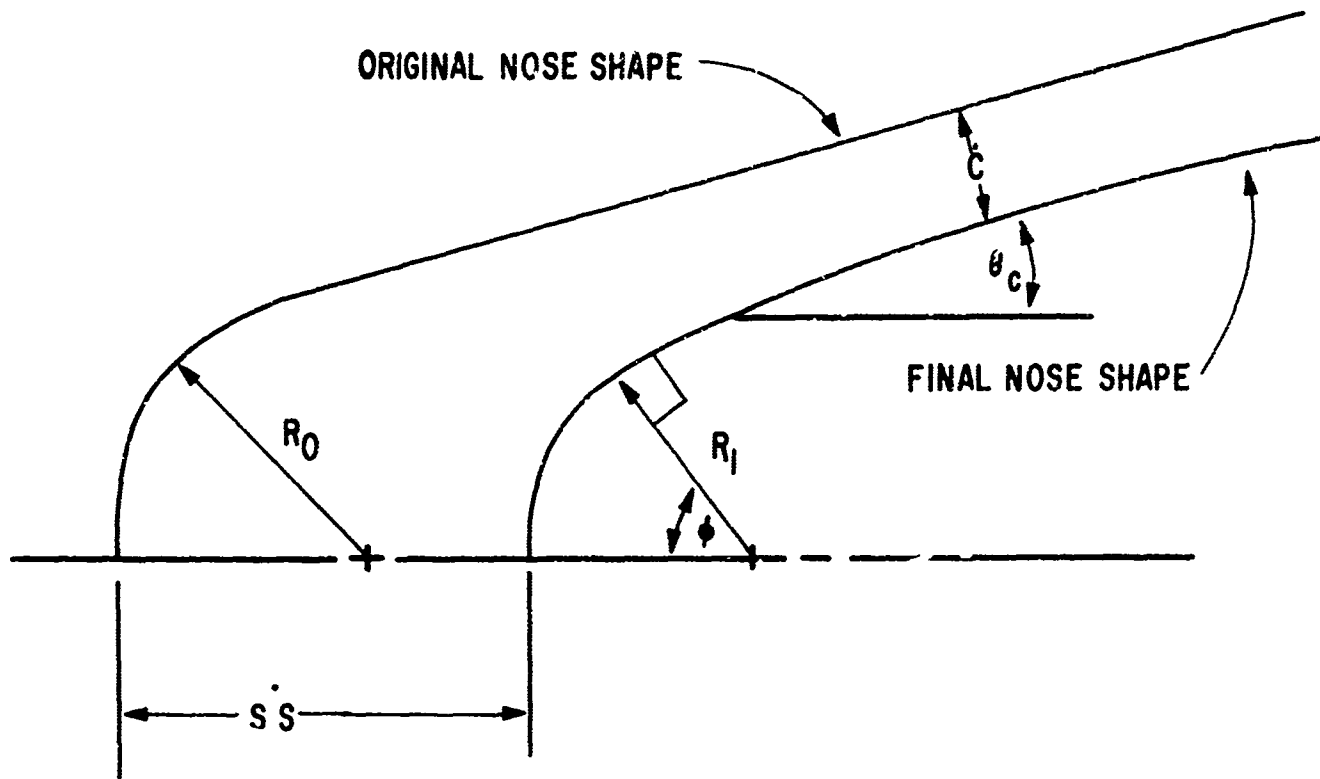


Figure 17. Vehicle Nose-Shape Change

$$R = R_0 + \frac{1}{(1 - \sin \theta_c)} \left[ \sin \theta_c \int_0^t \left( \frac{\dot{q}_c}{\rho Q^*} \right)_S dt - \int_0^t \left( \frac{\dot{q}_c}{\rho Q^*} \right)_C dt \right]$$

The integral represents the erosion at a given time during the flight for the locations at the stagnation point S and a point on the cone C at a location of wetted length of about five times the nose radius.

It is now possible to evaluate the final nose radius for a given application. Let

$$\dot{S}_S = \int_0^t \left( \frac{\dot{q}_c}{\rho Q^*} \right)_S dt \quad (115)$$

and

$$\dot{C} = \int_0^t \left( \frac{\dot{q}_c}{\rho Q^*} \right)_C dt \quad (116)$$

therefore

$$R = R_o + \frac{1}{(1 - \sin \theta_c)} \left[ \sin \theta_c \dot{S}_s - \dot{C} \right] \quad (117)$$

where

$R_o$  = original nose radius, ft

$R$  = instantaneous nose radius, ft

$\theta_c$  = cone half-angle

$\dot{S}_s$  = stagnation point recession rate, ft/sec

$\dot{C}$  = recession rate at location,  $C$ , ft/sec (see figure 17)

#### f. Boundary Layer Transition

The problem of transition from laminar to turbulent boundary layer flow has always been a troublesome one for the aeronautical engineer. The standard approach usually has been to design conservatively, that is, for turbulent flow. In general, the transition Reynolds number has been found to primarily depend upon the local Mach number, which has been observed in the analysis of boundary layer transition on a series of flight vehicles. Transition on blunt spherical nose vehicles with low boundary layer edge Mach numbers appears to occur at Reynolds number of  $1.5 \times 10^6$ , while on sharp bodies where the edge of the boundary layer edge Mach numbers approach 10 Reynolds numbers as high as  $2.0 \times 10^7$  have been observed prior to transition. Thus, for untested vehicle configurations, it is necessary to investigate the relationship of the Mach number and the Reynolds number in both the high and low Mach number regions. This relationship between Mach number and Reynolds number stands to reason if their definitions are understood. The parameter Mach number describes the influence of compressibility on heat transfer and flow phenomena and is defined



as the ratio of the gas or flight velocity to the local or ambient speed of sound. The Reynolds number, which describes the nature of flow, is a dimensionless measure of the ratio of inertial to viscous forces.

It should be noted that flight data on sharp bodies indicate that, generally, transition does not occur instantaneously over the entire vehicle so that it may travel several thousand feet between the onset of transition and the establishment of a fully turbulent boundary layer. See references 23 and 24.

The approach used in this program is to apply the sharp and blunt body transition Reynolds numbers simultaneously. Therefore, the sharp body criterion is described by the expression

$$Re_{TRAN} = 6.6 \times 10^7$$

and the blunt body criterion is applied when either of the following two Reynolds numbers are reached with conditions stated

$$Re_{TRAN} = 1.5 \times 10^6 \text{ on the spherical nose}$$

$Re_{TRAN} = 5.0 \times 10^6$  on the conical portion aft of the spherical nose for a wetted length of five times the nose radius.

The above criteria has been found to correlate with experimental data.

## 5. AERODYNAMICS

The one item that plays the most significant role in determining the performance of a high velocity or hypersonic reentry vehicle is aerodynamic drag. Experimental results have been correlated with theoretical studies of the nonsteady effects on a reentry vehicle or missile during its trajectory. These results indicate that the aerodynamic forces can be determined accurately by assuming quasi-steady flow conditions; that is, the flow field at any instant of time is the same as that associated with steady motion at the same velocity. Furthermore, it has been demonstrated that the boundary layer behaves in a quasi-steady manner. Therefore, assuming that the validity of these results carry over for an ablating blunt cone, the instantaneous drag coefficients can be obtained from equivalent steady-state conditions.

This program assumes that the drag consists of three components: cone pressure or forebody drag, base pressure drag, and skin friction or viscous drag. It is convenient to consider these quantities as distinct quantities which can be added together to obtain the total vehicle drag. Though all these quantities are distinct, some are dependent upon the boundary layer condition. For example, the condition of the boundary layer, laminar or turbulent, which influences the viscous drag can also influence the base drag.

#### a. Cone Pressure Drag

The forebody pressure drag for a slender body at zero angle-of-attack or at incidence can be calculated on the basis of the modified slender-body theory.

The forebody pressure drag is defined by the relation

$$D_p = - \iint_{S_m} P \cos(\eta, V_\infty) dS_m$$

where  $\cos(\eta, V_\infty)$  is the cosine of the angle between  $V_\infty$  and the outward normal to the vehicle surface. The area  $S_m$  comprises the total area of the vehicle except base area.

For a sharp cone at  $\alpha = 0$ , the pressure  $P_c$  is constant over the entire body, and the forebody drag is then only a function of the pressure ratio  $P_c/P_\infty$  across the conical shock. Solutions for this pressure ratio as a function of Mach number and cone angle,  $\theta_c$ , have been tabulated by Kopal (Ref. 25) for the ratio of specific heats of  $\gamma = 1.405$ . These results have been employed directly in the drag equation in order to obtain the forebody drag on a sharp cone. The analysis produced the equation

$$C_{Dp} = 4 \sin^2 \theta_c \left[ \frac{2.5 + 8 M_\infty \sin \theta_c}{1 + 16 M_\infty \sin \theta_c} \right] \quad (118)$$

which is similar to the form of the Newtonian drag equation for a sharp cone. The last term represents the variation from the Newtonian theory due to Mach number effects.

Nose blunting of a cone will increase the pressure drag. Local overpressures are induced near a blunt nose by the strong nose-shock curvature. The results of sharp and blunt body solutions have been correlated as a function of  $M_\infty$ ,  $\theta_c$ , and  $R_N/R_B$ . However, for bluntness ratios,  $R_N/R_B$  less than 0.05, the values for the drag due to the overpressures are low compared to  $C_{Dp}$ .

This program does account for the drag due to the nose being blunt within the limits expressed above. The sharp cone drag is modified by assuming the nose drag to be that of a sphere minus that of the cone replaced by the sphere. Because both of these terms are referenced to the projected area of the sphere, the area or bluntness ratio,  $R_N/R_B$ , term is needed as a correctness factor. Therefore, the pressure coefficient for a blunt cone is (Ref. 26)

$$C_{Dp_{\text{cone}}} = C_{Dp} + \left[ 0.58 - C_{Dp} \right] \left( \frac{R_N}{R_B} \right)^2 \quad (119)$$

where 0.58 is drag coefficient for a sphere obtained from reference 27

$C_{Dp}$  is from equation (118)

$R_N/R_B$  is the bluntness ratio of nose radius to cone base radius

#### b. Skin-Friction Drag

The skin-friction or viscous drag is defined as

$$D_V = \iint_{S_m} \tau \cos(\tau, V_\infty) dS_m$$

where  $\tau$  is the local skin friction per unit area due to viscosity, and  $\cos(\tau, V_\infty)$  is the cosine of the angle between  $V_\infty$  and the tangent to the vehicle surface in the  $\tau$  direction as shown in figure 18.

The skin-friction drag coefficient can be defined as

$$C_{Df} = \frac{2\pi}{A_B} \int_0^L C_{f_x} \cos \alpha \, r \, dx \quad (120)$$

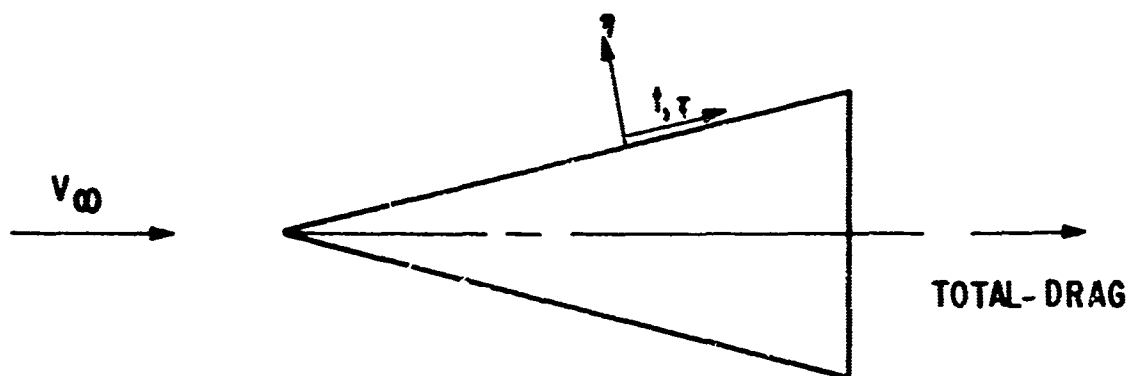


Figure 18. Aerodynamic Body Subject to Normal and Tangential Forces

where  $C_{f_\infty}$  is the local skin-friction coefficient defined as

$$C_{f_\infty} = \frac{\tau}{1/2 \rho V_\infty^2} \quad (121)$$

(1) Local Skin-Friction Coefficient

(a) Laminar Flow,  $\alpha = 0$

The local skin-friction coefficient is calculated using the Blasius flat plate in compressible solution. The flat plate solution was modified for conical flow by the Mangler transformation (Ref. 28) and for compressibility by Eckert's reference enthalpy method (Ref. 29).

The modified Blasius equation is

$$C_f = \frac{0.664}{\sqrt{Re_x}} \sqrt{3} \sqrt{\frac{\rho_e^* u_e^*}{\rho_e u_e} \left( \frac{\rho_e V_e^2}{\rho_\infty V_\infty^2} \right)} \quad (122)$$

where

$$Re_x = \frac{\rho_e V_e x}{\mu_e} \quad \text{boundary layer edge Reynolds number}$$

$x$  = distance along sharp cone surface

$\rho_e$  = density of air

$\mu_e$  = viscosity of air

$V_e$  = velocity

The superscript \* and subscripts e and  $\infty$  signify the property is evaluated at the reference enthalpy or conditions at the edge of boundary layer or free stream, respectively.

Equation (121) can be simplified to

$$C_{f_\infty} = \frac{1.15}{\sqrt{x}} \frac{V_e^{1.5} (\rho^* \mu^*)^{0.5}}{\rho_\infty V_\infty^2} \quad (123)$$

Letting

$$R_{\infty x} = \frac{\rho_\infty V_\infty x}{\mu_\infty}$$

and rearranging equation (123),

$$C_{f_\infty} = \frac{1.15}{\sqrt{R_{\infty x}}} \left( \frac{V_e}{V} \right)^{1.5} \sqrt{\frac{\rho^* \mu^*}{\rho_\infty \mu_\infty}} \quad (124)$$

From references 30 and 31, the quantity

$$\sqrt{\frac{\rho^* \mu^*}{\rho_\infty \mu_\infty}}$$

can be replaced by

$$\sqrt{\frac{\rho^* \mu^*}{\rho_\infty \mu_\infty}} = \left( \frac{p_c}{p_\infty} \right)^{0.5} \left( \frac{h^*}{h_\infty} \right)^{-0.185}$$

where

$$\frac{h^*}{h_\infty} = \frac{h^*}{h_e} \frac{T_e}{T_\infty}$$

It is assumed that cone pressure  $p_c$  is equal to boundary layer edge pressure,  $p_e$ , the free-stream specific heat is equal to the boundary layer edge specific heat, the specific heat ratio is  $\gamma = 1.405$ , and the recovery factor,  $R_F$ , is equal to 0.8426. Therefore, equation (123) for the local skin-friction coefficient can be written as

$$C_{f_\infty} = \frac{1.15}{\sqrt{R_{\infty x}}} \left( \frac{v_e}{v_\infty} \right)^{1.5} \left( \frac{p_e}{p_\infty} \right)^{0.5} \left( \frac{T_e}{T_\infty} \right)^{-0.185} \left( \frac{h^*}{h_e} \right)^{-0.185} \quad (125)$$

where

$$\frac{h^*}{h_e} = 0.5 + 0.5 \frac{Tw}{T_\infty} \left( \frac{T_e}{T_\infty} \right)^{-1} + 0.0374 M_e^2 \quad (126)$$

The ratios,

$$\left( \frac{v_e}{v_\infty} \right)$$

$$\left( \frac{p_e}{p_\infty} \right)$$

$$\left( \frac{T_e}{T_\infty} \right)$$

are obtained from conical flow results given by Bertram (Refs. 31 and 32) which have been correlated as a function of the hypersonic similarity parameter,  $M_\infty \sin \theta_c = K_c$ . A curve-fit of these results produced the following relations:

$$\frac{v_e}{v_\infty} = \left[ 1 - \frac{1.4}{M_\infty^2} (K_c)^{1.9} \right]^{0.5} \quad (127)$$

$$\frac{p_e}{p_\infty} = 1 + 2.8 K_c^2 \left[ \frac{2.5 + 8 K_c}{1 + 16 K_c} \right] \quad (128)$$

$$\frac{T_e}{T_\infty} = 1 + 0.0966 K_c + 0.2267 (K_c)^2 \quad (129)$$

The local Mach number can be computed from

$$\frac{M_e}{M_\infty} = \frac{V_e}{\sqrt{\gamma_e R_e T_e}} \cdot \frac{\sqrt{\gamma_\infty R_\infty T_\infty}}{V_\infty}$$

From previous assumptions and reducing

$$\frac{M_e}{M_\infty} = \frac{V_e}{V_\infty} \left( \frac{T_e}{T_\infty} \right)^{-0.5} \quad (130)$$

The local laminar skin-friction is now defined in terms of free-stream conditions, wall temperature, and cone angle.

(b) Turbulent Flow,  $\alpha = 0$

Using the Blasius flat plate incompressible solution modified in a similar manner for conical flow and compressibility, the Blasius equation for the local skin friction coefficient in turbulent flow is

$$C_{f_\infty} = \frac{0.0698}{R_{ex}^{0.2}} \left( \frac{\rho^*}{\rho_e} \right)^{0.8} \left( \frac{u^*}{u_e} \right)^{0.2} \frac{\rho_e V_e^2}{\rho_\infty V_\infty^2} \quad (131)$$

By substituting in a manner similar to laminar flow, and using the same ratios and rearranging, the following expression for the turbulent skin-friction coefficient in terms of free-stream conditions, wall temperature, and cone angle is given by

$$C_{f_\infty} = \frac{0.0698}{(R_{ex})^{0.2}} \left( \frac{V_e}{V_\infty} \right)^{1.8} \left( \frac{p_e}{p_\infty} \right)^{0.8} \left( \frac{h^*}{h_e} \right)^{-0.53} \left( \frac{T_e}{T_\infty} \right)^{-0.53} \quad (132)$$

where

$$\left( \frac{h^*}{h_e} \right) = 0.5 + 0.5 \left( \frac{T_w}{T_\infty} \right) \left( \frac{T_e}{T_\infty} \right)^{-1} + 0.0388 M_e^2$$

for  $R_F = 0.877$ .

(2) Average Skin-Friction Coefficient

To obtain the average skin-friction coefficient for both laminar and turbulent flow, the local value must be integrated over the cone surface. Therefore, substituting  $\pi R_B^2$  for  $A_B$  in equation (120) and rearranging

$$C_{Df} = \frac{2\pi}{\pi R_B^2} \int_0^L C_{f_\infty} \cos \theta_c (r) dx \quad (133)$$

but

$$C_{f_\infty} = \left( C_{f_\infty} \right)_{x=L} \left( \frac{L}{x} \right)^{0.5}$$

and

$$R_B^2 = L^2 \sin^2 \theta_c$$

$$r = x \sin \theta_c$$

Substituting these relations into equation (133), then

$$C_{Df} = \frac{2}{L^2 \sin^2 \theta_c} \left( C_{f_\infty} \right)_{x=L} (L)^{0.5} \cos \theta_c \sin \theta_c \int_0^L (x)^{0.5} dx \quad (134)$$

Now, combining equation (134) with equations (125) and (133), the average skin-friction coefficients for laminar and turbulent flow are, respectively,

$$C_{Df_L} = \frac{1.533}{\sqrt{R_{xL}}} \left( \frac{v_e}{v_\infty} \right)^{1.5} \left( \frac{p_e}{p_\infty} \right)^{0.5} \left( \frac{T_\infty}{T_e} \right)^{0.185} \left( \frac{h_e}{h^*} \right)^{0.185} (\cot \theta_c)$$

and

$$C_{Df_T} = \frac{0.0776}{\left( R_{xL} \right)^{0.7}} \left( \frac{v_e}{v_\infty} \right)^{1.2} \left( \frac{p_e}{p_\infty} \right)^{0.2} \left( \frac{T_\infty}{T_e} \right)^{0.2} \left( \frac{h_e}{h^*} \right)^{0.2} (\cot \theta_c) \quad (135)$$



The wall temperature used in the enthalpy ratio equation is taken as an average temperature over the entire vehicle surface.

#### c. Base Drag

The base pressure drag is determined by the mechanics of the wake, for which there is as yet no complete theory. Values of the base pressure coefficient  $C_{PB}$  must be obtained experimentally. Because of this, analytical methods of treating base drag are usually replaced by a correlation of experimental results. As the vehicle velocity increases, the base pressure, and therefore, the base-drag coefficient are affected similarly to other pressures. Once the speed is well into the supersonic region, most base pressures for a cone approach about 70 percent of a vacuum. Hoerner (Ref. 33) has correlated a wide variety of data for conical flow with the following result:

$$C_{PB} = 1.43/M_\infty^2 \quad (136)$$

for a vacuum. Reducing this by 70 percent

$$C_{PB} = 1.001/M_\infty^2 \quad (137)$$

for  $M \geq 2$ .

#### d. Stability Derivatives

The stability derivatives used in this program are for a sharp cone and are easily derived from references 34 and 35. These derivatives consisted of the damping coefficient per radian of pitch angle of attack,  $CMQ$ , and the rate of change of the normal force coefficient per degree of pitch angle of attack,  $CN_\alpha$ . These values are for the following cone conditions:

$$4.0^\circ < \alpha_c < 10^\circ$$

and

$$0 < R_N/R_B < 0.3$$

The rate of change of pitching moment coefficient,  $CMA$ , can be computed as

$$CMA = - (SN) (CNA) / DIA \quad (138)$$

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where

SM = static margin in feet

DIA = reference diameter in feet

Because of the symmetry of the conical vehicle used in this program, the values produced by yaw angle of attack and yaw angular velocities are equal in magnitude to pitch (data). Therefore,

$$C_{NR} = C_{NQ} \quad (139)$$

$$C_{NB} = -C_{NA} \quad (140)$$

$$C_{YB} = -C_{YA} \quad (141)$$

The derivatives are used to compute the aerodynamic forces and moments needed to restore the vehicle to zero angle of attack in pitch and yaw.

### SECTION III

#### SUBROUTINES

##### 1. ATMOSPHERIC PROPERTIES

The following list of subroutines are incorporated in STRAB-6 to compute the necessary atmospheric properties. This atmosphere uses the equations derived for the 1962 U.S. Standard atmosphere.

###### a. Subroutine RHOF

This subroutine computes the atmospheric density as a function of geometric altitude from the main program. The geometric altitude is converted to geopotential altitude in the subroutine and the density is computed accordingly. The units of density,  $\rho$ , are  $\text{lbs-sec}^2/\text{ft}^4$ .

###### b. Subroutine VSD

This subroutine calculates the velocity of sound,  $V_s$ , in feet/sec as a function of geometric altitude.

###### c. Subroutine TEMPA

The ambient temperature,  $TEMP$ , is calculated in this subroutine as a function of geometric altitude. The units are degrees R.

###### d. Subroutine VISCO

This subroutine computes the viscosity of air as a function of geometric altitude. The units are  $\text{lb-sec}/\text{ft}^2$ .

###### e. Subroutine PRESS

The ambient pressure,  $PE$ , is calculated in this subroutine as a function of geometric altitude. The units are  $\text{lb}/\text{ft}^2$ .

##### 2. HOT AIR PROPERTIES

###### a. Subroutine VISCOT

This subroutine calculates the viscosity of air as a function of temperature. It is used primarily for the hot air in the nonablating boundary layer. The units are  $\text{lb}/\text{sec-ft}$ .

b. Subroutine TCAT

The thermal conductivity, TCA, of hot air is calculated in this subroutine as a function of temperature. The units of TCA are Btu/ft-sec-°R.

3. ABLATOR THERMAL PROPERTIES

The following subroutines are used to compute the thermal properties of carbon phenolic, phenolic silica (Refrasil), and ATJ graphite. The properties computed are the thermal conductivity and the specific heat. The carbon phenolic and phenolic silica ablator have both virgin and char properties as well as densities. From all available data for ATJ graphite, it appears that no distinction is made between virgin or char conditions. The subroutines that compute the thermal properties for carbon phenolic and phenolic silica have the same names to minimize card changes in the main program.

a. Subroutine VTCPC

This subroutine computes the virgin thermal conductivity for carbon phenolic or for phenolic refrasil as a function of wall temperature. The units are Btu/ft-sec-°R.

b. Subroutine VCPPC

The virgin specific heat for carbon phenolic or phenolic silica is calculated by this subroutine as a function of wall temperature. The units are Btu/lb-°R.

c. Subroutine CTCPC

The char thermal conductivity for carbon phenolic or phenolic silica is computed by this subroutine as a function of wall temperature. The units are in Btu/ft-sec-°R.

d. Subroutine CCPP

This subroutine computes the char specific heat of carbon phenolic or phenolic silica as a function of wall temperature. The units are Btu/lb-°R.

4. ABLATION GAS PROPERTIES

Subroutines GCPB and GASTC compute the thermal properties of the ablative gas-air mixture in the boundary layer. These subroutines are used with the carbon phenolic ablator only. They compute the gas specific heat and gas thermal conductivity. The values associated with phenolic refrasil and ATJ graphite are inputted as single values.

## 5. STABILITY DERIVATIVES SUBROUTINES

These subroutines are incorporated into STRAB-6 for computation of the necessary aerodynamics discussed in section II-5. The equations are general for the following cone parameters:

$$4.0^\circ < \theta_c < 10.0^\circ$$

$$0 < R_N/R_B < 0.3$$

$$3.0 < M_\infty < 30$$

### a. Subroutine CDSF

This subroutine calculates the aerodynamic drag coefficients CP, CAN, CPB, and CDF as a function of  $M_\infty$  or Reynolds number. CDSF is called for in the main program.

#### (1) Subroutine CDFM

This subroutine computes the skin-friction drag, CDF, and is called into the program through subroutine CDSF. The boundary layer edge properties are calculated in this subroutine.

### b. Subroutine ACMQ

The damping coefficient, CMQ, is calculated in this subroutine as a function of free-stream Mach number. CMQ is per radian.

### c. Subroutine ACNA

This subroutine computes the normal force coefficient, CNA, as a function of free-stream Mach number. CNA is per radian.

## 6. SUBROUTINE INITIAL (Ref. 36)

This subroutine is incorporated into STRAB-6 to provide the necessary vehicle geometry calculations and changes in units. By reading in vehicle axial stations of interest, the subroutine will compute the transition Reynolds number, slant height (SL), smaller radius (RS), and larger radius (RL) for each conical segment. It will also compute the appropriate cone base radius and total slant height. See figure 16 for further clarification.

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7. SUBROUTINE ATTK AND SSLP

These subroutines are used to compute the complete range of angles of attack, ALP, and sideslip, BFT. These angles are computed as a function of the components of the relative velocity.

8. SUBROUTINE DLON

This subroutine computes the longitude of the vehicle with respect to the earth reference system. See section II-2c for derivation.

## SECTION IV

### PROGRAM OPERATION

#### 1. INPUT DATA REQUIREMENT

All input data that is subject to change either due to vehicle or trajectory requirements are read into STRAB-6 from the data statement. The word data is punched first, then the FORTRAN symbol, and then the numerical values. The symbols and numerical values are punched in Columns 7 through 72, inclusive, with up to nine continuation cards if needed. Columns 73 to 80, inclusive, can be used for comments or identification. The numerical values can be punched in any format except "I." Care should be taken to keep the symbol and its corresponding value in the same sequence. For example:

DATA V. AZN, NOFX, NAL, GAMMA/22414., 123.003, 8, 4, -39.92/

Also,

DATA WE, REQ, ET/0.729E-04, 20,927491E+06, 8.18133302E-02/

Another method of data card is

DATA (POS (I), I = 1, 6)/3.338, 15.189, 27.040, 38.851, 50.742, 65.556/

Another flexibility of the input routine, aside from the fact that the data can be introduced in any order of the symbols (with their associated values), is the feature that permits inputting the same symbol (and an associated value) again, thus superseding the previous value. This feature is convenient when basically constant vehicle configurations and initial conditions are maintained in many runs with only a few quantities changing. The "Standard" input data cards may be kept intact, and only the varying quantities may be punched on a card after the data cards which may be removed later.

For the heating portion of the program, there are several "Do Loops" that initialize temperatures, ablator lamina thickness, control indicators, and thermal properties of the adhesive bond and backface aluminum shell. Also, there are various equations to obtain initial trajectory conditions as well as initial dummy program starters.

a. Multi-Case Run

This program readily lends itself for a multi-case run with a minimal change. If the need arises for a parametric study of various vehicle geometry and trajectory missions, the following changes can be made to some of the data inputs and check for completion of the multi-case run.

```
Read 1500, NCASE, (POS(J), J=1, 8) 1500 FORMAT (I2, 7A10, A8)
```

```
Print 1501, NCASE, (POS(J), J=1, 8) 1501 FORMAT (1H1, 30X, 7A10, A8)
```

These read and print statements allow the identifier to be read into the program and case heading to be printed out before that particular case data output. The 1500 format allows the case number to be punched in the first two columns. When a zero is in these columns, the program will go to end with this check statement.

```
If (NCASE.NE.0) go to 1700  
END
```

where statement 1700 would be the return at the beginning of the program to start another case.

To update the vehicle configuration and trajectory parameters, the following data input is used

```
Read 1300, (POS(I), I=1,NOFX)
```

```
Read 1300, THETC, GAMMA, BI
```

```
Read 1300, V, AXN, GLAT, DLON
```

```
Read 1300, ALGTH, SNR 1300 Format (8F10.2)
```

All other data input cards would remain the same. See appendix II for multi-case listing and setup.

## 2. OUTPUT DATA REQUIREMENTS

The output quantities of the program and sample case are shown in appendixes I and III.

Before the printout of any results, a listing of the program and input data cards are printed. This record of the actual program listing and data input will assist in the identification of a run as well as an aid in troubleshooting in the event of arithmetic errors.



The first block of data printout of the matrix of temperatures, lamina thickness, etc., are obtained from the initialization "Do Loops" for heating portion of the program. This printout is done only once.

The second block of data is the trajectory output, aerodynamic coefficients, and boundary layer edge values. The next block of output is the wall and skin-profile temperatures, total recession, integrated weight loss rate, and total weight loss.

Printout is regulated by a time indicator. Data can be printed out at any time step desired. If a printout of data is wanted every 0.1 second, set  $BT = .1$ . The following method will then allow printout at that time step.

$$BTL = BT - H/6.$$

$$BTH = BT + H/6.$$

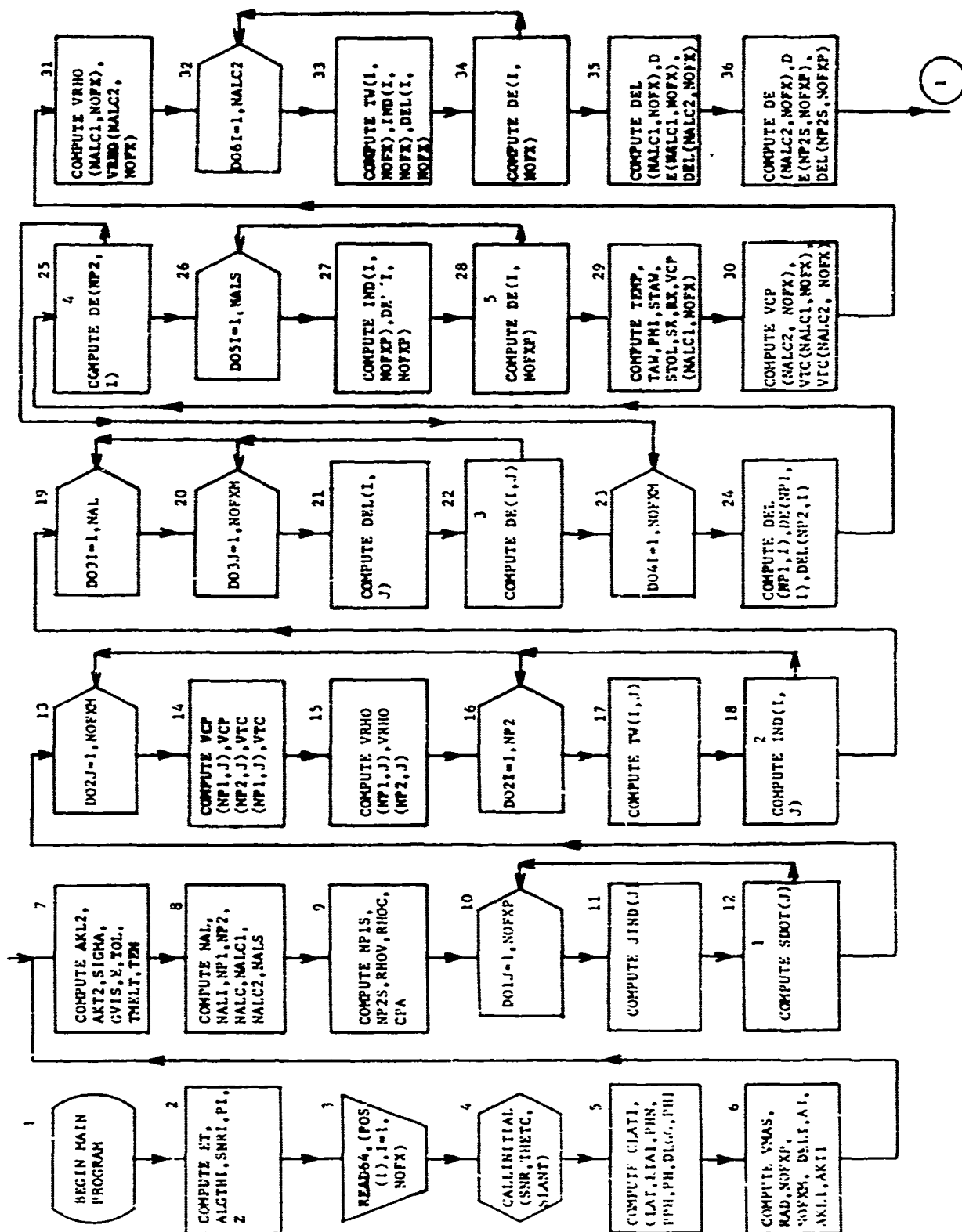
If  $(T .GT. BTL . AND. T. LT. BTH)$  go to (statement number)

$$BT = BT + .1$$

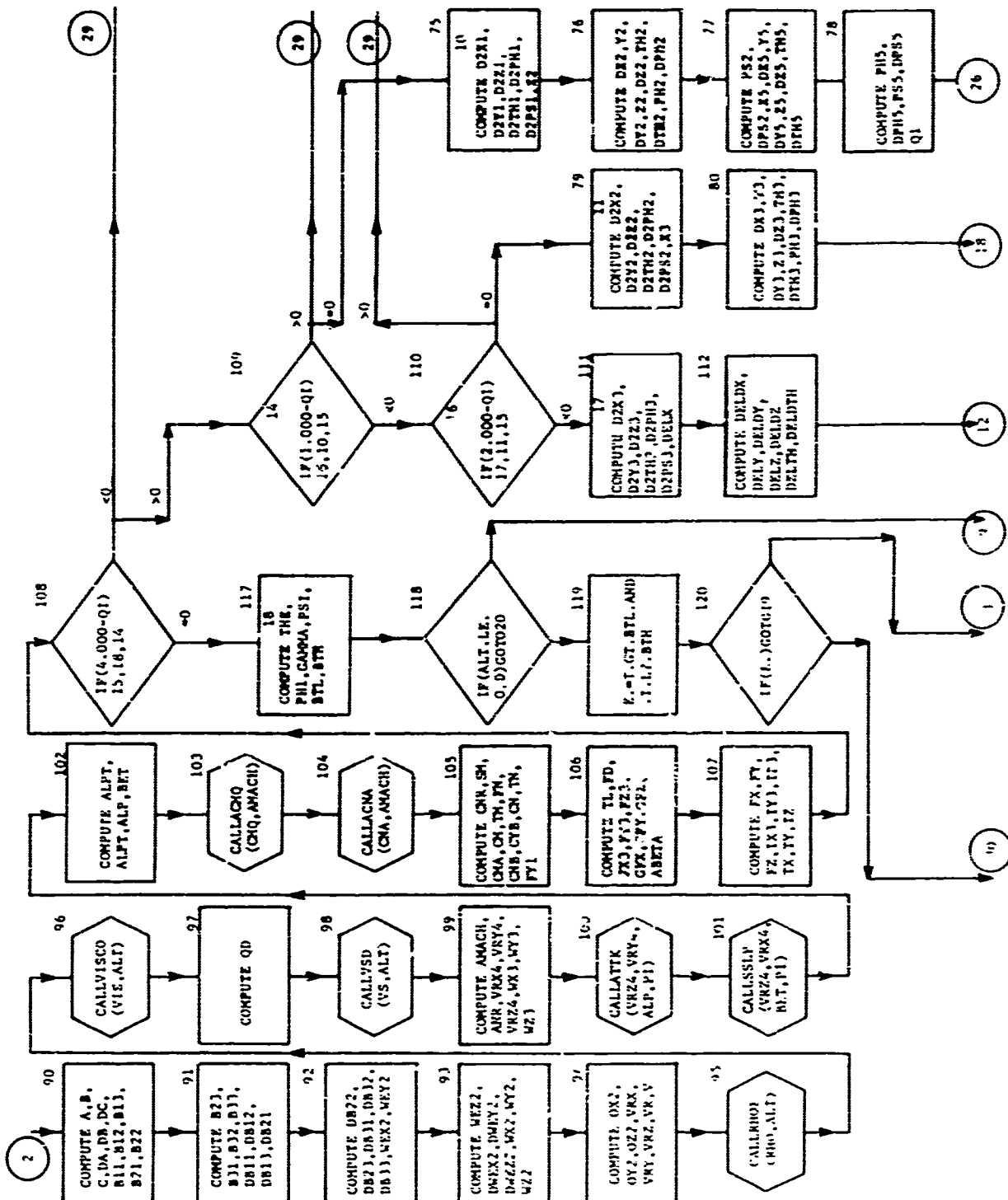
where H is the Runge-Kutta integration time step. This method will allow data printout and also go into the heating portion of the program.

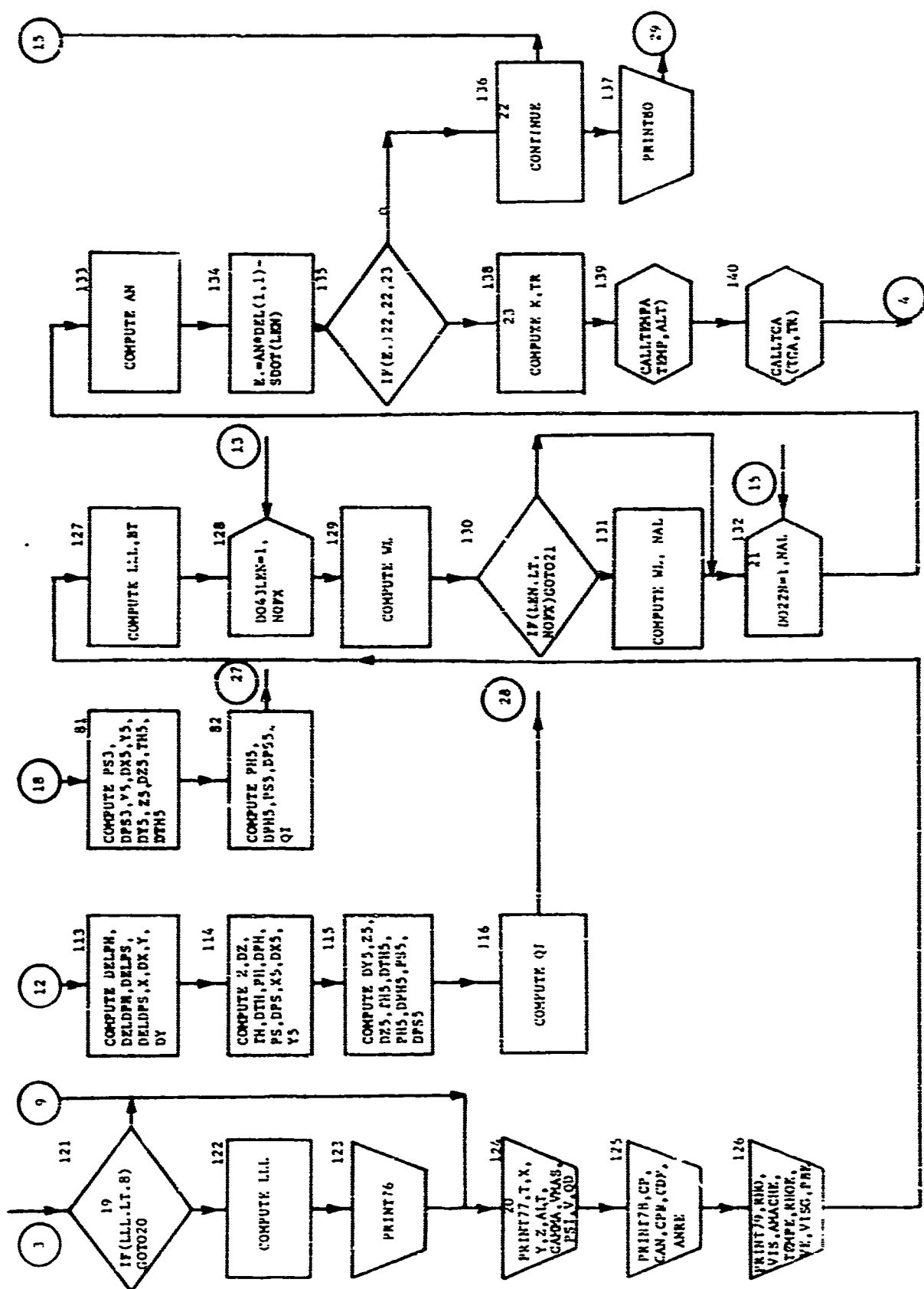
In addition to the printed output, selected quantities may also be obtained as punched output. The punched cards may be useful for machine plotting or as input to some other program.

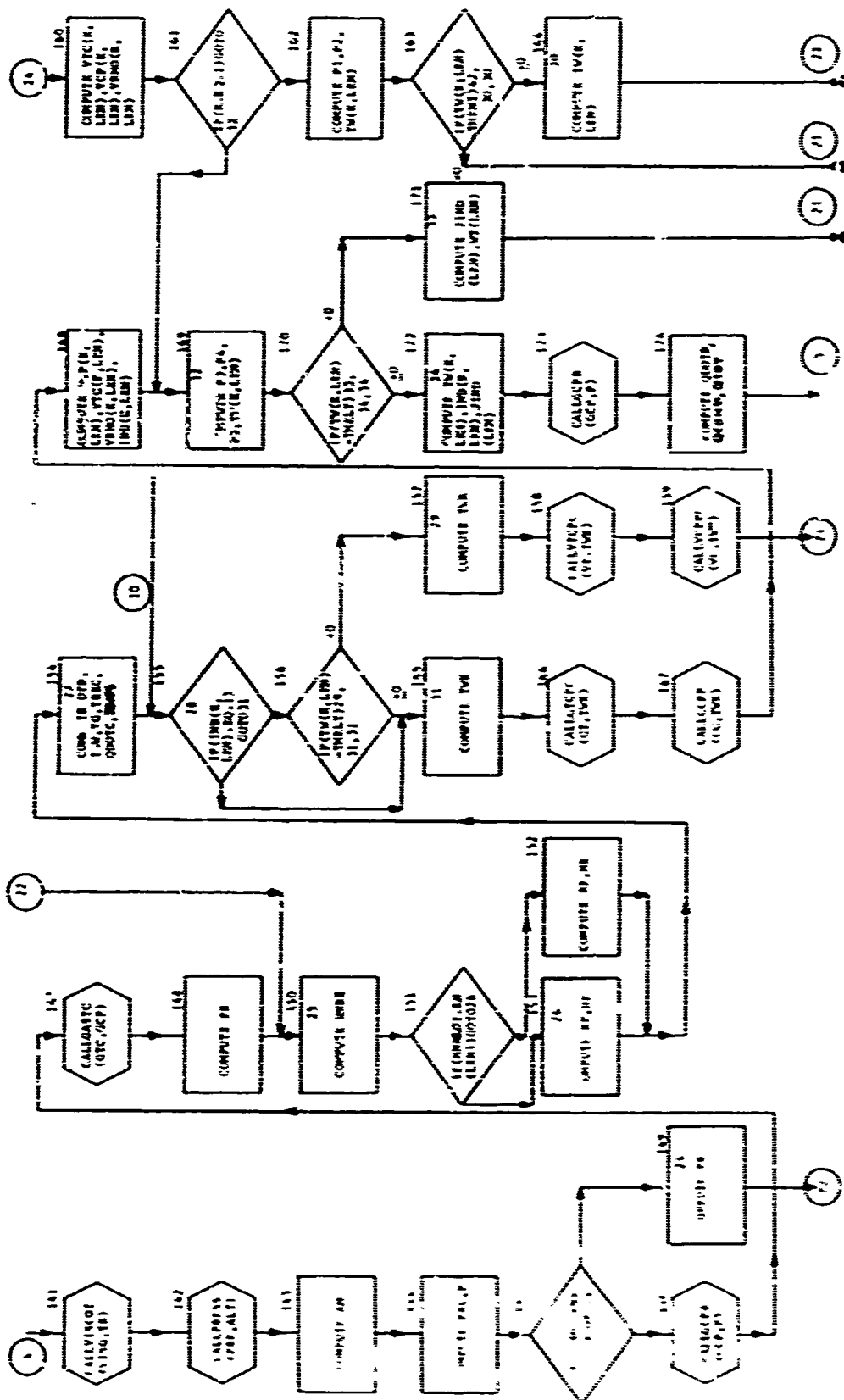
3. FLOW DIAGRAMS

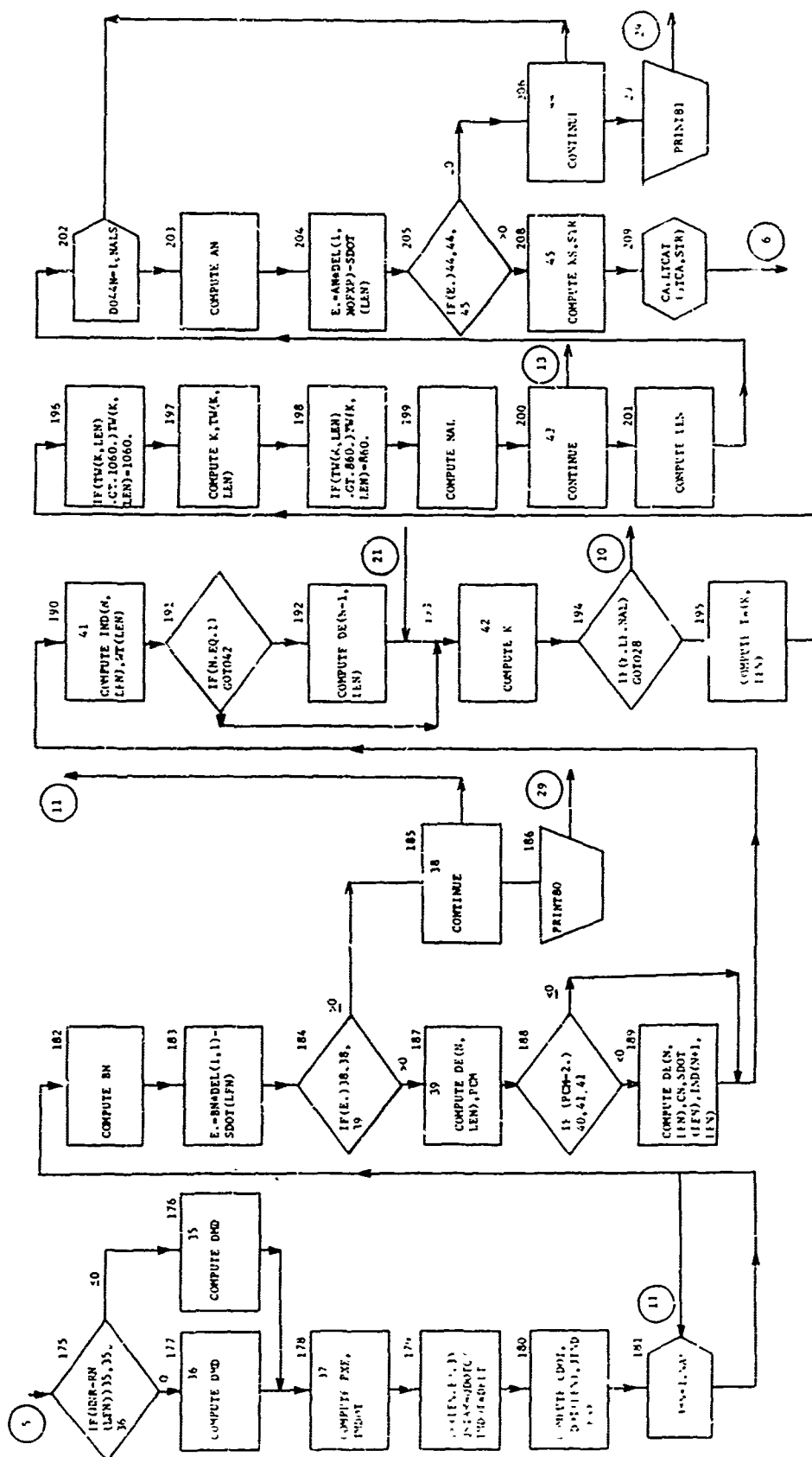


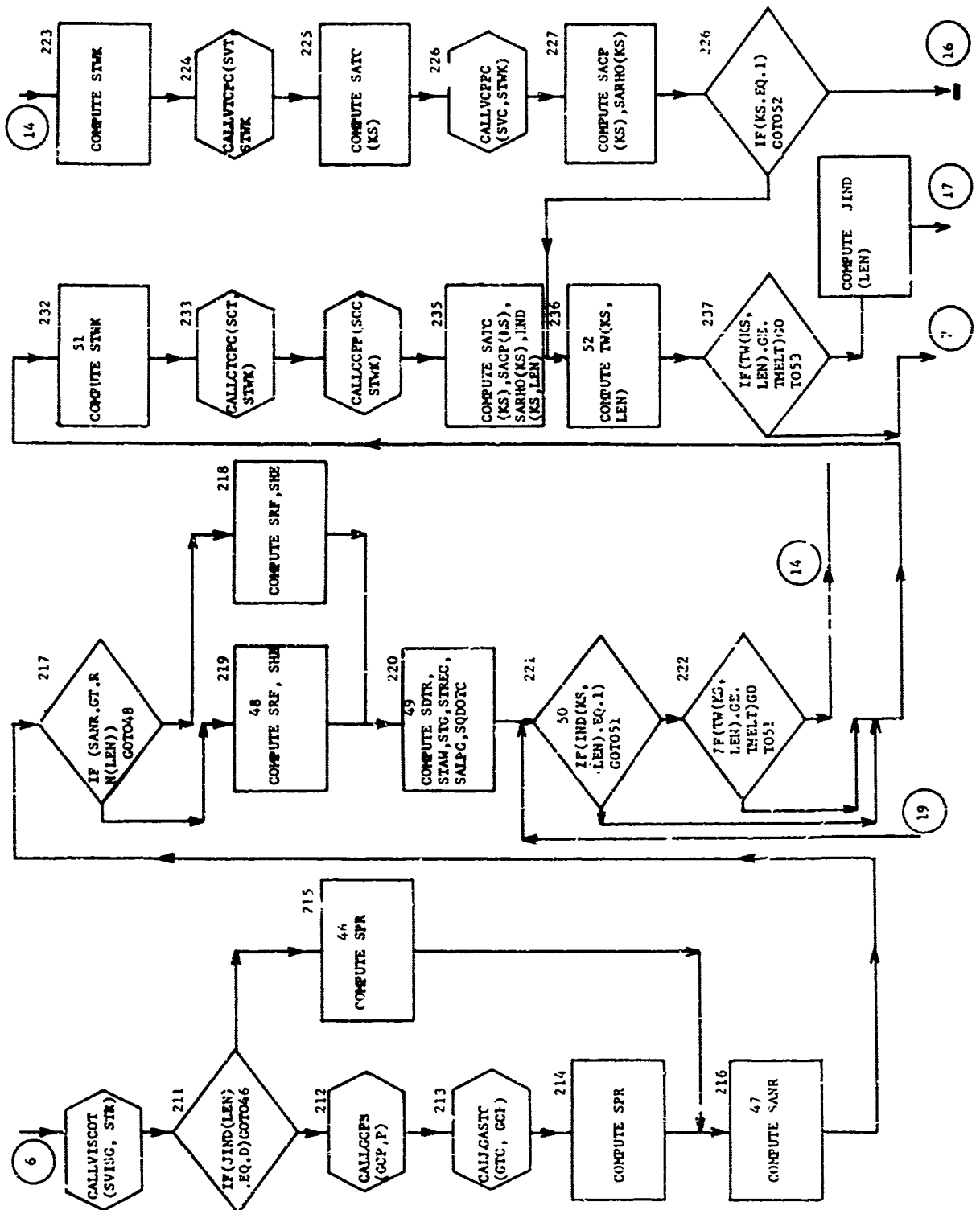




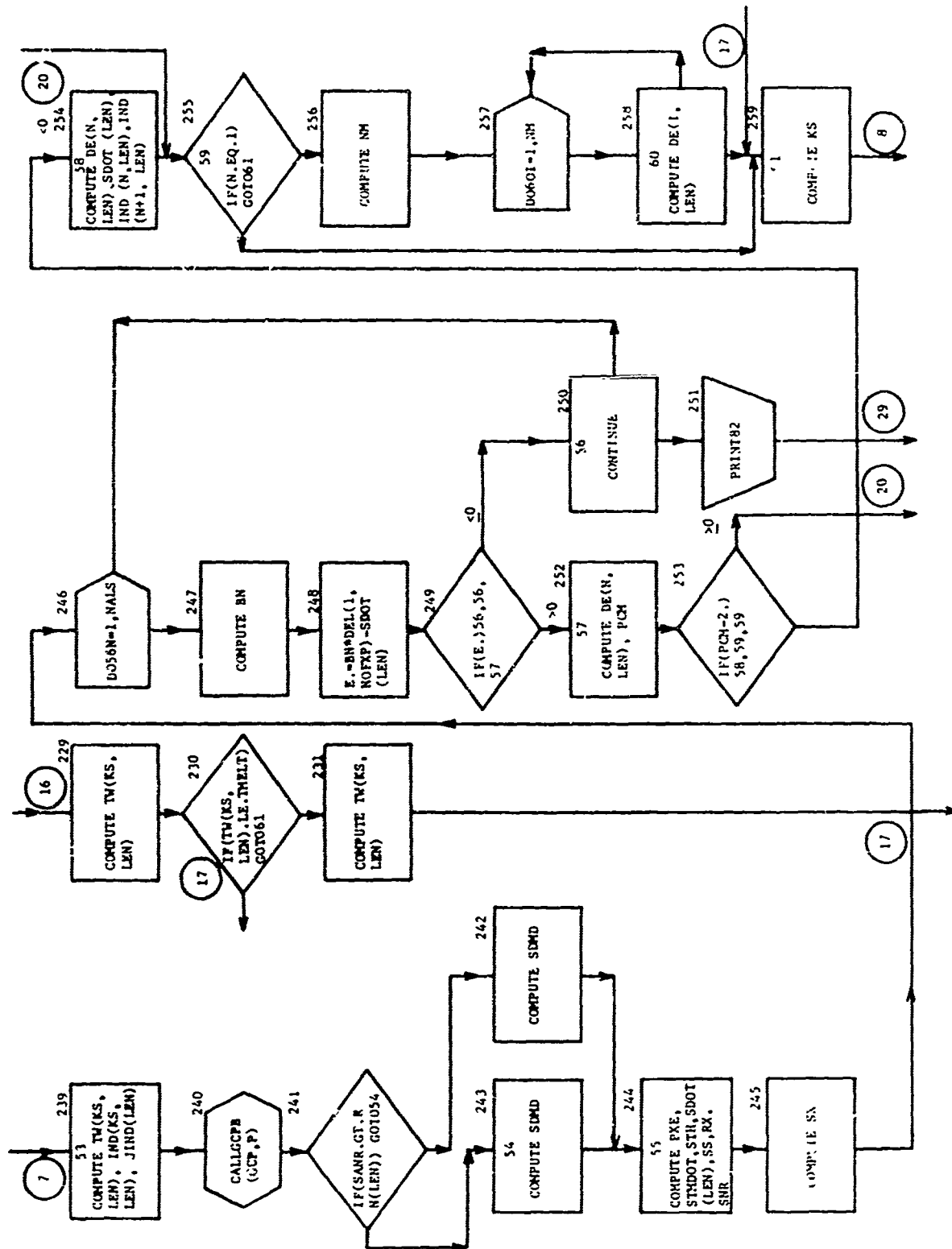


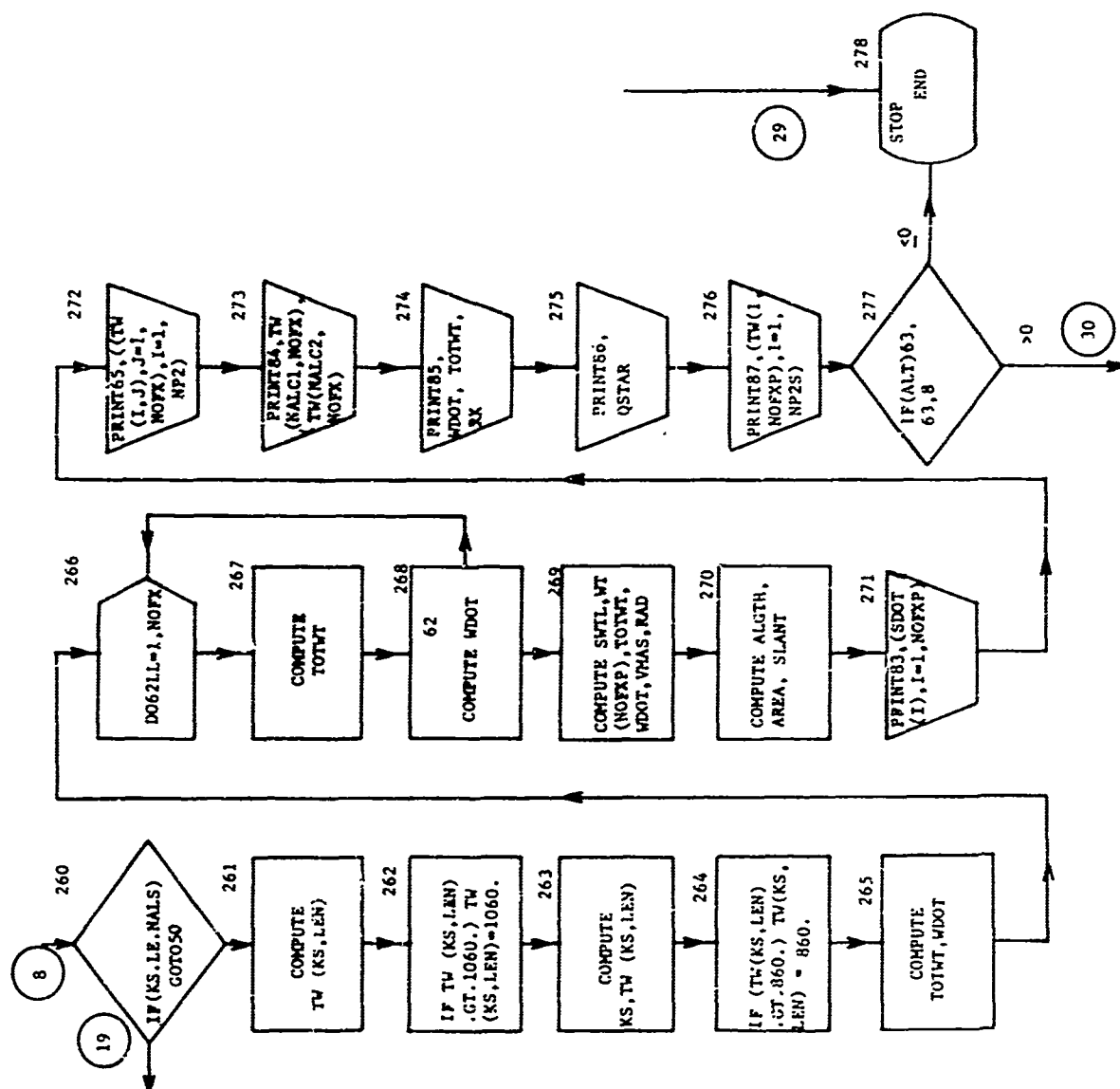












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APPENDIX I  
STRAB-6 PROGRAM LISTING

	PROGRAM STRAB2 (INPUT,OUTPUT)		
C	6 DEGREE TRAJECTORY WITH ABLATION HEATING AND STAG	A	2
	COMMON /A/ RE	A	3
	COMMON /B/ AREA,RAD,SLANT,SNR,THETC,V	A	4
	COMMON /C/ CPM,CP,CAN,CPR,CDF	A	5
	COMMON /D/ Tw(50,12),IND(50,12),JIND(12),VCP(50,12),VTC(50,12),VRH	A	6
	10(50,12),DEL(50,12),DE(50,12),SDOT(12),XI(12),RS(12),RL(12),SOL(12	A	7
	2),WT(12),TM(12),TWT(12),SACP(50),SATC(50),SARHO(50),RN(12),QSTR(12	A	8
	3),QDOTC(12),QTOT(12),QCOND(12)	A	9
	COMMON /E/ NOFXM,NAL,ALT,AMACH,QD,AMACHE,TEMPE,RHOE,VE,VISG,PBE,AN	A	10
	1RE	A	11
	COMMON /F/ NOFX,ALGTH,RADI,POS(11),AZN,GLAT	A	12
C	VEHICLE DATA	A	13
C	IF NOFX IS CHANGED CHANGE FORMAT 103	A	14
C	IF CONC ANGLE IS CHANGED SUB CDSF MUST BE CHANGED	A	15
	DATA SNR,THETC,RI,SM/.0208,.07853,6.27836,.59718/	A	16
	DATA PI,E,AJ,GRAV,G/3.14159,2.7183,778,.32,174,32.174/	A	17
	DATA REQ,PE,ET,WZ/20.927491E+06,.673852E-02,.08,81233,0.7292E-04/	A	18
	DATA NOFX,NAL,NALC,NALS/8,4,10,35/	A	19
	DATA DELT,OT,H,RT,AT/.05,.0005,.001,.05,1./	A	20
	DATA AZN,GLAT,DLON,GAMMA/154,.33,996,-107.5,-22.39/	A	21
	DATA V,Z,ALPHA/15167,.2,50E+03,7./	A	22
	DATA (POS(I),I=1,8)/6.607,18.570,30.533,42.496,54.459,66.422,78.38	A	23
	15.86,469/	A	24
	AIZ=0.51	A	25
	ATX=23.08	A	26
	CALL INITIAL (SNR,THETC,SLANT)	A	27
	ALGTHI=ALGTH	A	28
	SNRI=SNR	A	29
	CLAT1=ATAN((1.-ET**2)*TAN(GLAT))	A	30
	CLAT=CLAT1*180./PI	A	31
	ETA1=GLAT-CLAT1	A	32
	PHN=(90.-GAMMA-ALPHA)*PI/180.	A	33
	PPH=0.0	A	34
	PH=(1.+PPH)*PHN	A	35
	DLGG=DLON	A	36
	PHI=PH	A	37
	VMAC=81	A	38
	RAD=RADI	A	39
	NOFXP=NOFX+1	A	40
	NOFXM=NOFX-1	A	41
	AKL1=5370.	A	42
	AKT1=4240.	A	43
	AKL2=5.37	A	44
	AKT2=5.77	A	45
	GVIS=4.429E-5	A	46
	TOL=.00854	A	47
	TMELT=6760.	A	48
	TEM=544.	A	49
	NAL1=NAL	A	50
	NP1=NAL+1	A	51
	NP2=NAL+2	A	52
	NALC1=NALC+1	A	53
	NALC2=NALC+2	A	54
	NP1S=NALS+1	A	55
	NP2S=NALS+2	A	56
	RHCV=90.4	A	57
	RHCC=74.	A	58
	ATJRH0=120.96	A	59
	CPA=.2395	A	60
	DO 1 J=1,NOFXP	A	61

	JIND(J)=0	A 62
1	Soot(J)=0.	A 63
	DO 2 J=1,NOFXM	A 64
	VCP(NP1,J)=.315	A 65
	VCP(NP2,J)=.208	A 66
	VTC(NP1,J)=.74E-04	A 67
	VTC(NP2,J)=.03694	A 68
	VRHO(NP1,J)=91.7	A 69
	VRHO(NP2,J)=169.	A 70
	DO 2 I=1,NP2	A 71
	TW(I,J)=TEM	A 72
2	IND(I,J)=0	A 73
	DO 3 I=1,NAL	A 74
	DO 3 J=1,NOFXM	A 75
	DEL(I,J)=TOL	A 76
3	DE(I,J)=TOL	A 77
	DO 4 I=1,NOFXM	A 78
	DEL(NP1,I)=.00333	A 79
	DE(NP1,I)=.00333	A 80
	DEL(NP2,I)=.005	A 81
4	DE(NP2,I)=.005	A 82
	DO 5 I=1,NALS	A 83
	IND(I,NOFXP)=0	A 84
	DEL(I,NOFXP)=TOL	A 85
5	DE(I,NOFXP)=TOL	A 86
	TEMP=298.2	A 87
	TAW=TEMP	A 88
	PHI=80.967	A 89
	STAW=TEMP	A 90
	STOL=TOL	A 91
	SX=.01745*RS(NOFXP)*PHI	A 92
	RX=SNR	A 93
	VCP(NALC1,NOFX)=.315	A 94
	VCP(NALC2,NOFX)=.208	A 95
	VTC(NALC1,NOFX)=.74E-04	A 96
	VTC(NALC2,NOFX)=.03694	A 97
	VRHO(NALC1,NOFX)=91.7	A 98
	VRHO(NALC2,NOFX)=169.	A 99
	DO 6 I=1,NALC2	A 100
	TW(I,NOFX)=TEM	A 101
	IND(I,NOFX)=0	A 102
	DEL(I,NOFX)=TOL	A 103
6	DE(I,NOFX)=TOL	A 104
	DEL(NALC1,NOFX)=.00333	A 105
	DE(NALC1,NOFX)=.00333	A 106
	DEL(NALC2,NOFX)=.005	A 107
	DE(NALC2,NOFX)=.005	A 108
	DE(NP2S,NOFXP)=.005	A 109
	DEL(NP2S,NOFXP)=.005	A 110
	DE(NP1S,NOFXP)=.00333	A 111
	DEL(NP1S,NOFXP)=.00333	A 112
	SACP(NP1S)=.315	A 113
	SACP(NP2S)=.208	A 114
	SATC(NP1S)=.74E-04	A 115
	SATC(NP2S)=.03694	A 116
	SARHO(NP1S)=91.7	A 117
	SARHO(NP2S)=169.	A 118
	DO 7 N=1,NP2S	A 119
7	TW(N,NOFXP)=TEM	A 120
	PSI=0.	A 121
	DAZ=0.0	A 122
	AZ=(1.000+DAZ)*AZN	A 123

SGLAT=SIN(GLAT)	A 124
CGLAT=COS(GLAT)	A 125
SAZ=SIN(AZ)	A 126
CAZ=COS(AZ)	A 127
WEX=-WE*CGLAT*SAZ	A 128
WEY=WE*CGLAT*CAZ	A 129
WEZ=WE*SGLAT	A 130
RE1=REQ/(1.000*PE*(SIN(CLAT1))**2)**.500	A 131
RE=RE1	A 132
RXM=RE1*SIN(ETA1)*SAZ	A 133
RYM=-RE1*SIN(ETA1)*CAZ	A 134
RZM=RE1*COS(ETA1)	A 135
T=0.0	A 136
X=0.0	A 137
DX=0.0	A 138
Y=0.0	A 139
PPH=0.0	A 140
PH=(1.0+PPH)*PHN	A 141
DZ=V*SIN(GAMMA*PI/180.)	A 142
DY=V*COS(GAMMA*PI/180.)	A 143
LLL=0	A 144
ALT=Z	A 145
TH=0.0	A 146
DTH=0.0	A 147
DPH=0.0	A 148
PS=0.0	A 149
DELPS=0.0	A 150
DPSN=7.*PI	A 151
DPS=(1.+DELPS)*DPSN	A 152
U=0.1407639E+17	A 153
BJ=1623.41E-06	A 154
BH=6.04E-06	A 155
BK=6.37E-06	A 156
AREA=PI*RAD**2	A 157
OJA=1.781	A 158
DCD=0.00	A 159
D2X=0.0	A 160
D2Y=0.0	A 161
D2Z=0.0	A 162
D2TH=0.0	A 163
D2PH=0.0	A 164
D2PS=0.0	A 165
PRINT 71	A 166
PRINT 72. (JIND(I),I=1,NOFXP)	A 167
PRINT 92. (SDOT(I),I=1,NOFXP)	A 168
PRINT 73. ((IND(I,J),J=1,NOFX),I=1,NALC2)	A 169
PRINT 74. ((VCP(I,J),J=1,NOFX),I=1,NALC2)	A 170
PRINT 75. ((VTC(I,J),J=1,NOFX),I=1,NALC2)	A 171
PRINT 76. ((VRM(I,J),J=1,NOFX),I=1,NALC2)	A 172
PRINT 77. ((DEL(I,J),J=1,NOFX),I=1,NALC2)	A 173
PRINT 78. ((DE(I,J),J=1,NOFX),I=1,NALC2)	A 174
PRINT 79. ((TW(I,J),J=1,NOFX),I=1,NALC2)	A 175
PRINT 71	A 176
PRINT 80	A 177
CALL VSD (VS,ALT)	A 178
CALL RHOF (RHO,ALT)	A 179
AMACH=V/VS	A 180
CALL VISCO (VIS,ALT)	A 181
ANR=RHO*V*SLANT/VIS	A 182
CALL CDSF (CD,AMACH,ANR)	A 183
QN=.5*RHO*V**2	A 184
ALPT=ABS(ALPHA)	A 185

	PRINT B1, T,X,Y,Z,ALT,GAMMA,VHAS,PSI,V,OD,AMACH,CD,ANR,PH1,THE,ALP	A 186
	1,BET,PS,ALPT,XE,YE,ZE,FD,FN,GFZ,CLAT,DLOH,ABETA,GCIR,AIZ,AIX	A 187
	LLL=LLL*1	A 188
	GO TO 9	A 189
8	D2X=FX/VHAS-2.*DA-C*WEY+B*WEZ*GX	A 190
	D2Y=FY/VHAS-2.*DB-A*WEZ+C*WEY*GY	A 191
	D2Z=FZ/VHAS-2.*DC-B*WEX+A*WEY*GZ	A 192
	D2TH=TY/AIX-WX2*OZ2+AIZ*WZ2*OX2/AIX-DWEY2	A 193
	D2PH=(-TX/AIX+AIZ*WZ2*OY2/AIX-WY2*OZ2*DPH*DTM*SIN(TH)+DWEY2)/COS(TH)	A 194
	D2PS=TZ/AIZ-AIX*WY2*OX2/AIZ+AIX*WX2*OY2/AIZ+D2PH*SIN(TH)+DPH*DTM*C	A 195
	10S(TH)-DWEZ2	A 196
	CALL CDSF (CD,AMACH,ANR)	A 197
	PER=1.*.0375*ALPT+.00156*ALPT**2.	A 198
	CD=CD*PER	A 199
9	X1=X+DX*H/2.	A 200
	DX1=DX+D2X*H/2.	A 201
	Y1=Y+DY*H/2.	A 202
	DY1=DY+D2Y*H/2.	A 203
	Z1=Z+DZ*H/2.	A 204
	DZ1=DZ+D2Z*H/2.	A 205
	TH1=TH+DTM*H/2.	A 206
	DTM1=DTM+D2TH*H/2.	A 207
	PH1=PH+DPH*H/2.	A 208
	DPH1=DPH+D2PH*H/2.	A 209
	PS1=PS+DPS*H/2.	A 210
	DPS1=DPS+D2PS*H/2.	A 211
	X5=X1	A 212
	DX5=DX1	A 213
	Y5=Y1	A 214
	DY5=DY1	A 215
	Z5=Z1	A 216
	DZ5=DZ1	A 217
	TH5=TH1	A 218
	DTM5=DTM1	A 219
	PH5=PH1	A 220
	DPH5=DPH1	A 221
	PS5=PS1	A 222
	DPS5=DPS1	A 223
	Q1=1.000	A 224
	GO TO 12	A 225
10	D2X1=FX/VHAS-2.*DA-C*WEY+B*WEZ*GX	A 226
	D2Y1=FY/VHAS-2.*DB-A*WEZ+C*WEY*GY	A 227
	D2Z1=FZ/VHAS-2.*DC-B*WEX+A*WEY*GZ	A 228
	D2TH1=TY/AIX-WX2*OZ2+AIZ*WZ2*OX2/AIX-DWEY2	A 229
	D2PH1=(-TX/AIX+AIZ*WZ2*OY2/AIX-WY2*OZ2*DPH5*DTM5*STH5+DWEY2)/CTH5	A 230
	D2PS1=TZ/AIZ-AIX*WY2*OX2/AIZ+AIX*WX2*OY2/AIZ+D2PH1*STH5+DPH5*DTM5*	A 231
	1CTH5-DWEZ2	A 232
	X2=X+DX1*H/2.	A 233
	DX2=DX+D2X1*H/2.	A 234
	Y2=Y+DY1*H/2.	A 235
	DY2=DY+D2Y1*H/2.	A 236
	Z2=Z+DZ1*H/2.	A 237
	DZ2=DZ+D2Z1*H/2.	A 238
	TH2=TH+DTM1*H/2.	A 239
	DTM2=DTM+D2TH1*H/2.	A 240
	PH2=PH+DPH1*H/2.	A 241
	DPH2=DPH+D2PH1*H/2.	A 242
	PS2=PS+DPS1*H/2.	A 243
	DPS2=DPS+D2PS1*H/2.	A 244
	X5=X2	A 245
	DX5=DX2	A 246
		A 247



	Y5=Y2	A 248
	DY5=DY2	A 249
	Z5=Z2	A 250
	DZ5=DZ2	A 251
	TH5=TH2	A 252
	DTM5=DTM2	A 253
	PH5=PH2	A 254
	DPH5=DPH2	A 255
	PS5=PS2	A 256
	DPS5=DPS2	A 257
	QI=2.000	A 258
	GO TO 13	A 259
11	D2X2=FX/VMAS-2.*DA-C*WEY*B*WEZ*GX	A 260
	D2Y2=FY/VMAS-2.*DB-A*WEZ*C*WEX*GY	A 261
	D2Z2=FZ/VMAS-2.*DC-B*WEX*A*WEY*GZ	A 262
	D2TH2=TY/AIX-WX2*OZ2+AIZ*WZ2*OX2/AIX-DWEY2	A 263
	D2PH2=(-TX/AIX+AIZ*WZ2*OY2/AIX-WY2*OZ2+DPH5*DTM5*STM5*DWEX2)/CTH5	A 264
	D2PS2=TZ/AIZ-AIX*WY2*OX2/AIZ+AIW*WX2*OY2/AIZ-D2PH2*STM5*DPH5*DTM5	A 265
	1CTH5=DWEZ2	A 266
	X3=X+DX2*H/2.	A 267
	DX3=DX+D2X2*H/2.	A 268
	Y3=Y+DY2*H/2.	A 269
	DY3=DY+D2Y2*H/2.	A 270
	Z3=Z+DZ2*H/2.	A 271
	DZ3=DZ+D2Z2*H/2.	A 272
	TH3=TH+DTM2*H/2.	A 273
	DTM3=DTM+D2TM2*H/2.	A 274
	PH3=PH+DPH2*H/2.	A 275
	DPH3=DPH+D2PH2*H/2.	A 276
	PS3=PS+DPS2*H/2.	A 277
	DPS3=DPS+D2PS2*H/2.	A 278
	X5=X3	A 279
	DX5=DX3	A 280
	Y5=Y3	A 281
	DY5=DY3	A 282
	Z5=Z3	A 283
	DZ5=DZ3	A 284
	TH5=TH3	A 285
	DTM5=DTM3	A 286
	PH5=PH3	A 287
	DPH5=DPH3	A 288
	PS5=PS3	A 289
	DPS5=DPS3	A 290
	QI=3.000	A 291
12	T=DT	A 292
13	R=SQRT((RXH*X5)**2+(RYH*Y5)**2+(RZH*Z5)**2)	A 293
	ZE=(RZH*Z5)*SGLAT-CGLAT*((RXH*X5)*SAZ-(RYH*Y5)*CAZ)	A 294
	AL=ASIN(ZE/R)	A 295
	CAL=COS(AL)	A 296
	SAL=SIN(AL)	A 297
	CLAT=AL*180./PI	A 298
	RE=REQ/(1.000+PE*SAL**2)**.500	A 299
	ALT=R-RE	A 300
	YE=(RXH*X5)*CAZ+(RYH*Y5)*SAZ	A 301
	XE=(RZH*Z5)*CGLAT+SGLAT*((RXH*X5)*SAZ-(RYH*Y5)*CAZ)	A 302
	CALL LONG (DLG,XE,YE,PI)	A 303
	CLG=COS(DLG)	A 304
	SLG=SIN(DLG)	A 305
	DLOG=DLGG+DLG*180./PI	A 306
	GCIR=((RE*RE)/2.)*ACOS(SAL*SIN(CLAT1)+CAL*COS(CLAT1)*COS(AL-CLAT1)	A 307
	111	A 308
	G11=-SGLAT+CAL*CLG+CGLAT*SAL	A 309

G12=SGLAT*CAL*CLG-CGLAT*CAL	A 310
G21=-CAL*SLG	A 311
G22=SAL*SLG	A 312
G31=-CGLAT*CAL*CLG-SGLAT*SAL	A 313
G32=-CGLAT*SAL*CLG-SGLAT*CAL	A 314
P12=1.-3.*SAL**2	A 315
P13=3.*SAL-5.*SAL**3	A 316
P14=3.-30.*SAL**2+35.*SAL**4	A 317
P15=SIN(2.*AL)	A 318
P6=CAL*(1.-5.*SAL**2)	A 319
P7=SAL*CAL*(-3.-7.*SAL**2)	A 320
GR=U/R**2*(1.-9J*(REQ/R)**2*P12+.4./5.*BH*(REQ/R)**5+.13*BK/6.*(REQ/R)**4*P14)	A 321
SL=U/R**2*(-DJ*(REQ/R)**2*P15+.3./5.*RH*(REQ/R)**3*P6+.2./3.*BK*(REQ/R)**4*P7)	A 322
SL=U/R**2*(-DJ*(REQ/R)**2*P15+.3./5.*RH*(REQ/R)**3*P6+.2./3.*BK*(REQ/R)**4*P7)	A 323
GX=GR*(G11*SAZ-G21*CAZ)*GL*(G12*SAZ-G22*CAZ)	A 324
GY=GR*(G21*SAZ-G11*CAZ)*GL*(G22*SAZ-G12*CAZ)	A 325
GZ=GR*G31*GL*G32	A 326
CP55=COS(P55)	A 327
SP55=SIN(P55)	A 328
STH5=SIN(TH5)	A 329
CTH5=COS(TH5)	A 330
SPH5=SIN(PH5)	A 331
CPH5=COS(PH5)	A 332
A11=CP55*CTH5	A 333
A12=SP55*CPH5-STH5*SPH5*CP55	A 334
A13=-CP55*STH5*CPH5-SP55*SPH5	A 335
A21=-SP55*CTH5	A 336
A22=SP55*STH5*SPH5*CP55*CPH5	A 337
A23=SP55*STH5*CPH5-SPH5*CP55	A 338
A31=STH5	A 339
A32=CTH5*SPH5	A 340
A33=CTH5*CPH5	A 341
A=WEY*(RZH*Z5)-(RYH*Y5)*WEZ	A 342
B=WFZ*(RXH*X5)-(RZH*Z5)*WEX	A 343
C=WEX*(RYH*Y5)-(RXH*X5)*WEY	A 344
DA=DZ5*WEY-DY5*WEZ	A 345
DB=DX5*WEZ-DZ5*WEX	A 346
DC=DY5*WEX-DX5*LEY	A 347
B11=CTH5	A 348
B12=-STH5*SPH5	A 349
B13=-STH5*CPH5	A 350
B21=0.	A 351
B22=CPH5	A 352
B23=-SPH5	A 353
B31=STH5	A 354
B32=CTH5*SPH5	A 355
B33=CTH5*CPH5	A 356
DB11=-DTH5*STH5	A 357
DB12=-DTH5*CTH5*SPH5-DPH5*CPH5*STH5	A 358
DB13=-DTH5*CTH5*CPH5-DPH5*SPH5*STH5	A 359
DB21=0.	A 360
DB22=-DPH5*SPH5	A 361
DB23=-DPH5*CPH5	A 362
DB31=DTH5*CTH5	A 363
DB32=DPH5*CPH5*CTH5-DTH5*STH5*SPH5	A 364
DB33=-DTH5*STH5*CPH5-DPH5*SPH5*CTH5	A 365
WEX2=B11*WEX+B12*WEY+B13*WEZ	A 366
WEY2=B21*WEX+B22*WEY+B23*WEZ	A 367
WEZ2=B31*WEX+B32*WEY+B33*WEZ	A 368
DWEX2=DB11*WEX+DB12*WEY+DB13*WEZ	A 369
DWEY2=DB21*WEX+DB22*WEY+DB23*WEZ	A 370
	A 371

DWE72=DR31\*WEX+DS32\*WEY+DR33\*WEZ

WX2=WFY2-DPH5\*CTH5

WY2=WEY2-DTH5

WZ2=WFZ2-DPS5-DPH5\*STH5

OX2=WX2

OY2=WY2

OZ2=WEZ2-DPH5\*STH5

VRX=OX5

VRY=OY5

VRZ=OZ5

VR=SQRT(VRX\*\*2+VRY\*\*2+VRZ\*\*2)

V=SQRT(OX5\*\*2+OY5\*\*2+OZ5\*\*2)

CALL RHOF (RHO,ALT)

CALL VISCO (VIS,ALT)

QD=.5\*RHO\*V\*\*2

CALL VSD (VS,ALT)

AMACH=V/VS

ANR=RHO\*V\*SLANT/VIS

VRX4=A11\*VRX+A12\*VRY+A13\*VRZ

VRY4=A21\*VRX+A22\*VRY+A23\*VRZ

VRZ4=A31\*VRX+A32\*VRY+A33\*VRZ

WX3=-DPH5\*CTH5\*CP55-DTH5\*SP55

WY3=DPH5\*CTH5\*SP55-DTH5\*CP55

WZ3=DPS5-DPH5\*STH5

CALL ATTK (VRZ4,VRY4,ALP,PI)

CALL SSLP (VRZ4,VRX4,BET,PI)

ALPT=ALP\*(ALP)

ALPT=ALPT\*180./PI

ALP=ALP\*180./PI

BET=BET\*180./PI

ALP1=ABSF(ALP)

BET1=ABSF(BET)

CALL ACMQ (CMQ,AMACH)

CALL ACNA (CNA,AMACH)

CNA PER DEGREE

CMQ PER RADIAN

CNR=CMQ

CMA=-SM\*CNA/DIA

CM=CMA\*ALP+CMQ\*WX3\*DIA/(2.\*V)

TM=CM\*QD\*AREA\*DIA

FN=CNA\*QD\*AREA\*ALP

CNB=-CMA

CYB=-CNA

CN=CNR\*BET+CNR\*WY3\*DIA/(2.\*V)

TN=CN\*QD\*AREA\*DIA

FY1=CYB\*QD\*AREA\*BET

TL=0.

FD=CD\*QD\*AREA

FX3=FY1

FY3=-FN

FZ3=-FD

GFX=FX3/(VMAS\*GRAV)

GFY=(FY3-VMAS\*GRAV\*SIN(PH))/(VMAS\*GRAV)

GFZ=(FZ3+VMAS\*GRAV\*COS(PH))/(VMAS\*GRAV)

ABETA=VMAS\*GRAV/(CD\*AREA)

FX=A11\*FX3+A21\*FY3+A31\*FZ3

FY=A12\*FX3+A22\*FY3+A32\*FZ3

FZ=A13\*FX3+A23\*FY3+A33\*FZ3

TX3=TM

TY3=TN

TZ3=TL

TX=TX3\*CP55-TY3\*SP55

A 372

A 373

A 374

A 375

A 376

A 377

A 378

A 379

A 380

A 381

A 382

A 383

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A 422

A 423

A 424

A 425

A 426

A 427

A 428

A 429

A 430

A 431

A 432

A 433

	TY=TX3*SPS5+TY3*CPS5	A 434
	TZ=TZ3	A 435
	IF (4.000-QI) 15,18,14	A 436
14	IF (1.000-QI) 16,10,15	A 437
15	GO TO 69	A 438
16	IF (2.000-QI) 17,11,15	A 439
17	D2X3=FX/VMAS-2.*DA-C*WEY+H*WEZ+GX	A 440
	D2Y3=FY/VMAS-2.*DB-A*WEZ+C*WEX+GY	A 441
	D2Z3=FZ/VMAS-2.*DC-B*WEX+A*WEY+GZ	A 442
	D2TH3=TY/AIX-WX2*OZ2+AIZ*WZ2*OX2/AIX-DWEY2	A 443
	D2PH3=(-TX/AIX+AIZ*WZ2*OY2/AIX-WY2*OZ2+DPH5*DTH5+STH5+DWEX2)/CTH5	A 444
	D2PS3=TZ/AIZ-AIX*WY2*OX2/AIZ+AIW*WX2*OY2/AIZ+D2PH3*STH5+DPH5*DTH5*	A 445
	1CTH5-DWEZ2	A 446
	DELX=H/6.*(DX+2.*DX1+2.*DX2+DX3)	A 447
	DELDX=H/6.*(D2X+2.*D2X1+2.*D2X2+D2X3)	A 448
	DELY=H/6.*(DY+2.*DY1+2.*DY2+DY3)	A 449
	DELDY=H/6.*(D2Y+2.*D2Y1+2.*D2Y2+D2Y3)	A 450
	DELZ=H/6.*(DZ+2.*DZ1+2.*DZ2+DZ3)	A 451
	DELDZ=H/6.*(D2Z+2.*D2Z1+2.*D2Z2+D2Z3)	A 452
	DELTH=H/6.*(DTH+2.*DTH1+2.*DTH2+DTH3)	A 453
	DELDTH=H/6.*(D2TH+2.*D2TH1+2.*D2TH2+D2TH3)	A 454
	DELPH=H/6.*(DPH+2.*DPH1+2.*DPH2+DPH3)	A 455
	DELDPH=H/6.*(D2PH+2.*D2PH1+2.*D2PH2+D2PH3)	A 456
	DELPS=H/6.*(DPS+2.*DPS1+2.*DPS2+DPS3)	A 457
	DEL DPS=H/6.*(D2PS+2.*D2PS1+2.*D2PS2+D2PS3)	A 458
	X=X+DELX	A 459
	DY=Y+DELY	A 460
	DY=Y+DELY	A 461
	DY=DY+DELY	A 462
	Z=Z+DELZ	A 463
	DZ=DZ+DELDZ	A 464
	TH=TH+DELTH	A 465
	DTH=DTH+DELDTH	A 466
	PH=PH+DELPH	A 467
	DPH=DPH+DELDPH	A 468
	PS=PS+DELPS	A 469
	DPS=DPS+DEL DPS	A 470
	X5=X	A 471
	DX5=DX	A 472
	Y5=Y	A 473
	DY5=DY	A 474
	Z5=Z	A 475
	DZ5=DZ	A 476
	TH5=TH	A 477
	DTH5=DTH	A 478
	PH5=PH	A 479
	DPH5=DPH	A 480
	PS5=PS	A 481
	DPS5=DPS	A 482
	QI=4.000	A 483
	GO TO 13	A 484
18	THE=TH*180./PI	A 485
	PH1=PH*180./PI	A 486
	GAMMA=ATAN(DZ5/DY5)*180./PI	A 487
	PSI=ATAN(DX5/DY5)*180./PI	A 488
	BTL=BT-M/4.	A 489
	BTH=BT+M/4.	A 490
	IF (ALT.LE.0.0) GO TO 20	A 491
	IF (T.GT.BTL.AND.T.LT.BTH) GO TO 19	A 492
	GO TO 8	A 493
19	BT=BT+.05	A 494
C	*****	A 495

C	*****	A 496
20	WL=0.0	A 497
	DO 49 LEN=1,NOFX	A 498
	WL=WL+SCL(LEN)	A 499
	IF (LEN.LT.NOFX) GO TO 21	A 500
	WL=.25	A 501
	NAL=NALC	A 502
21	DO 22 N=1,NAL	A 503
	AN=N	A 504
	IF (AN*DEL(1,1)-SDOT(LEN)) 22,22,23	A 505
22	CONTINUE	A 506
	PRINT 84	A 507
	GO TO 69	A 508
23	K=N	A 509
	IF (LEN.GE.NOFX) TMELT=6400.	A 510
	TR=TEMPE+.58*(TW(K,LEN)-TEMPE)+.19*(TAW-TEMPE)	A 511
	CALL TEMPA (TEMP,ALT)	A 512
	CALL TCAT (TCA,TR)	A 513
	CALL VISCOT (VISE,TEMPE)	A 514
	CALL VISCOT (VISG,TR)	A 515
	CALL PRESS (PRE,ALT)	A 516
	AM=AMACH	A 517
	PBL=PRE*(1+.2.8*AM**2*(SIN(THETC))**2*(2.5+8.*AM*SIN(THETC)))/(1.+1	A 518
	16.*AM*SIN(THETC)))	A 519
	P=PBL/2116.2	A 520
	IF (JIND(LEN).EQ.0) GO TO 24	A 521
	*****	A 522
	CALL GCPB (GCP,P)	A 523
	IF (LEN.GE.NOFX) GCP=.4	A 524
	*****	A 525
	CALL GASTC (GTC,GCP)	A 526
	PR=GCP*GVIS/GTC	A 527
	GO TO 25	A 528
24	PR=CPA*VISG/TCA	A 529
25	HNRF=RHOE*VE*WL/VISE*G	A 530
	RHOS=PBE/(1716.*TR)	A 531
	ANRR=RHOS*VE*WL/VISG*G	A 532
	IF (HNRF.GT.RN(LEN)) GO TO 26	A 533
	RF=PR*.5	A 534
	HE=.575*TCA/WL*PR*.33*HNRF*.5*(VISG/VISE*RHOS/RHOE)**.5	A 535
	GO TO 27	A 536
26	RF=PR*.33	A 537
	HE=.0296*TCA/WL*PR*.33*HNRF*.8*(VISG/VISE)**.2*(RHOS/RHOE)**.8	A 538
27	DTR=VE**2/(2.*G*AJ*CPA)	A 539
	TAW=TEMPE+RF*DTR	A 540
	QDOTC(LEN)=HE*(TAW-TW(K,LEN))	A 541
28	IF (IND(K,LEN).EQ.1) GO TO 33	A 542
	IF (TW(K,LEN)-TMELT) 29,33,33	A 543
29	TWK=TW(K,LEN)	A 544
	IF (LEN.GE.NOFX) GO TO 30	A 545
	CALL VTCPC (VT,TWK)	A 546
	CALL VCPPC (VC,TWK)	A 547
	VRHO(K,LEN)=RHOV	A 548
	GO TO 31	A 549
30	CALL TCATJ (VT,TWK)	A 550
	CALL CPATJ (VC,TWK)	A 551
	VRHO(K,LEN)=ATJRH0	A 552
31	VTC(K,LEN)=VT	A 553
	VCP(K,LEN)=VC	A 554
	IF (K.EQ.1) GO TO 36	A 555
	P1=VTC(K-1,LEN)*(TW(K-1,LEN)-TW(K,LEN))/(VRHO(K-1,LEN)*VCP(K-1,LEN	A 556
	1)*DE(K-1,LEN)**2)	A 557

	P2=VTC(K,LEN)*(TW(K,LEN)-TW(K+1,LEN))/(VRHO(K,LEN)*VCP(K,LEN)*DE(K	A 558
	1,LEN)**2)	A 559
	TW(K,LEN)=TW(K,LEN)+DELT*(P1-P2)	A 560
	IF (TW(K,LEN)-TMELT) 48,32,32	A 561
32	TW(K,LEN)=TMELT-1.	A 562
	GO TO 48	A 563
33	TWK=TW(K,LEN)	A 564
	IF (LEN.GE.NOFX) GO TO 34	A 565
	CALL CTCPC (CT,TWK)	A 566
	CALL CCPP (CC,TWK)	A 567
	VRHO(K,LEN)=RHOC	A 568
	GO TO 35	A 569
34	CALL TCATJ (CT,TWK)	A 570
	CALL CPATJ (CC,TWK)	A 571
	VRHO(K,LEN)=ATJRHOC	A 572
35	VCP(K,LEN)=CC	A 573
	VTC(K,LEN)=CT	A 574
	IND(K,LEN)=1	A 575
36	P3=DELT/(VCP(K,LEN)*VRHO(K,LEN)*DE(K,LEN))*2.	A 576
	P4=HE*(TAW-TW(K,LEN))	A 577
	P5=VTC(K,LEN)*(TW(K,LEN)-TW(K+1,LEN))/DE(K,LEN)	A 578
	HR=CPA*TMPE+RF*VE**2/(2.*G*AJ)	A 579
	QCOND(LEN)=-VTC(K,LEN)*(TW(K,LEN)-TW(K+1,LEN))/DE(K,LEN)	A 580
	TW(K,LEN)=TW(K,LEN)+P3*(P4-P5)	A 581
	IF (TW(K,LEN)-TMELT) 37,38,38	A 582
37	JIND(LEN)=0	A 583
	WT(LEN)=0.	A 584
	GO TO 48	A 585
38	TW(K,LEN)=TMELT	A 586
	IND(K,LEN)=1	A 587
	JIND(LEN)=1	A 588
	CALL GCPB (GCP,P)	A 589
	IF (LEN.GE.NOFX) GCP=.4	A 590
	QDOTR=0.	A 591
	QTOT(LEN)=QDOTC(LEN)+QCOND(LEN)	A 592
	IF (QTOT(LEN).LE.0.0) GO TO 48	A 593
	IF (LEN.LT.NOFX) GO TO 39	A 594
	DELH=HR-CPA*TMELT	A 595
	QSTAR=2000.*2.*DELH	A 596
	CDOT=QTOT(LEN)/(QSTAR*ATJRHOC)	A 597
	GO TO 43	A 598
39	IF (HNRE-RN(LEN)) 40,40,41	A 599
40	DMD=(QTOT(LEN))/(AKL1+AKL2*(HR-CPA*TMELT))	A 600
	GO TO 42	A 601
41	DMD=(QTOT(LEN))/(AKT1+AKT2*(HR-CPA*TMELT))	A 602
42	PXE=11.05E-04/TMELT	A 603
	TMDOT=DMD*(1.+2.64E+09/(PBL**2.67E**PXE))	A 604
	CDOT=TMDOT/RHOV	A 605
43	SDOT(LEN)=SDOT(LEN)+CDOT*DELT	A 606
	JIND(LEN)=1	A 607
	DO 44 N=1,NAL	A 608
	BN=N	A 609
	IF (BN*DEL(1,1)-SDOT(LEN)) 44,44,45	A 610
44	CONTINUE	A 611
	PRINT 84	A 612
	GO TO 69	A 613
45	DE(N,LEN)=DEL(N,LEN)-(SDOT(LEN)-(BN-1.)*DEL(1,LEN))	A 614
	PCM=VCP(N,LEN)*VRHO(N,LEN)*DE(N,LEN)**2/(DELT*VTC(N,LEN))	A 615
	IF (PCM-2.) 46,47,47	A 616
46	DE(N,LEN)=0.	A 617
	CN=N	A 618
	SDOT(LEN)=CN*TOL	A 619

	IND(N+1,LEN)=1	A 620
47	IND(N,LEN)=1	A 621
	WT(LEN)=PI*(RL(LEN)*RS(LEN)-2.*SDOT(LEN))*CDOT*RHOC*SOL(LEN)	A 622
	IF (N.EQ.1) GO TO 48	A 623
	DE(N-1,LEN)=0.	A 624
48	K=K+1	A 625
	IF (K.LE.NAL) GO TO 28	A 626
	TW(K,LEN)=TW(K,LEN)+DELT*(VTC(K-1,LEN)/(VRHO(K-1,LEN)*VCP(K-1,LEN)	A 627
	1*DE(K-1,LEN)**2)*(TW(K-1,LEN)-TW(K,LEN))-VTC(K,LEN)/(VRHO(K,LEN)*V	A 628
	2CP(K,LEN)*DEL(K,LEN)**2)*(TW(K,LEN)-TW(K+1,LEN)))	A 629
	IF (TW(K,LEN).GT.1060.) TW(K,LEN)=1060.	A 630
	K=K+1	A 631
	TW(K,LEN)=TW(K,LEN)+DELT*(VTC(K-1,LEN)/(VRHO(K-1,LEN)*VCP(K-1,LEN)	A 632
	1*DEL(K-1,LEN)**2)*(TW(K-1,LEN)-TW(K,LEN)))	A 633
	IF (TW(K,LEN).GT.860.) TW(K,LEN)=860.	A 634
	NAL=NALI	A 635
49	CONTINUE	A 636
	*****	A 637
	LEN=NOFXP	A 638
	DO 50 N=1,NALS	A 639
	AN=N	A 640
	IF (AN*DEL(1,NOFXP)-SDOT(LEN)) 50,50,51	A 641
50	CONTINUE	A 642
	PRINT 85	A 643
	GO TO 69	A 644
51	KS=N	A 645
	TMELT=6400.	A 646
	STR=TEMP+.58*(TW(KS,LEN)-TEMP)+.19*(STAW-TEMP)	A 647
	CALL TCAT (STCA,STR)	A 648
	CALL VISCOT (SVISG,STR)	A 649
	IF (JIND(LEN).EQ.0) GO TO 52	A 650
	*****	A 651
	CALL GCPB (GCP,P)	A 652
	IF (LEN.GE.NOFX) GCP=.4	A 653
	*****	A 654
	CALL GASTC (GTC,GCP)	A 655
	SPR=GCP*GVIS/GTC	A 656
	GO TO 53	A 657
52	SPR=CPA*SVISG/STCA	A 658
53	SFR=SPR*.5	A 659
	CALL PRESS(PRE,ALT)	A 660
	PTS=PRE*((1.-2.*AMACH**2)**3.5)*((6.0/(7.0*AMACH**2-1.0))**.25)	A 661
	RHOWS=PTS**2*(1.-2.*AMACH**2)**3.5	A 662
	DV=1./SN**2*(1.-2.*AMACH**2)**3.5	A 663
	GTAB=((1.-2.*AMACH**2)**3.5*(1.0-1.0/(1.4*AMACH**2))**.25	A 664
	SQDOTC=0.4*3*GTAB*SQRT(RHOWS*SVISG*DVG**2)/(2.*G*AJ)	A 665
55	SDTR=V**2/(2.*G*AJ*CPA)	A 666
	STAW=TEMP+SFR*SDTR	A 667
	STG=TEMP+SDTR	A 668
	STREC=TEMP+SFR*(STG-TEMP)	A 669
	SHE=SQDOTC/(STG-TW(KS,LEN))	A 670
56	IF (IND(KS,LEN).EQ.1) GO TO 57	A 671
	IF (TW(KS,LEN).GE.TMELT) GO TO 57	A 672
	STWK=TW(KS,LEN)	A 673
	CALL TCATJ (SYT,STWK)	A 674
	CALL CPATJ (SVC,STWK)	A 675
	SARHO(KS)=ATJRHQ	A 676
	SATC(KS)=SVT	A 677
	SACP(KS)=SVC	A 678
	IF (KS.EQ.1) GO TO 58	A 679
	TW(KS,LEN)=TW(KS,LEN)+DELT*(SATC(KS-1)/(SARHO(KS-1)*SACP(KS-1)*DE	A 680
	1*KS-1,LEN)**2)*(TW(KS-1,LEN)-TW(KS,LEN))-SATC(KS)/(SARHO(KS)*SACP(K	A 681

	2S)*DE(KS,LEN)**2)*(TW(KS,LEN)-TW(KS+1,LEN)))	A 682
	IF (TW(KS,LEN).LE.TMELT) GO TO 65	A 683
	TW(KS,LEN)=TMELT-1.	A 684
	GO TO 65	A 685
57	STWK=TW(KS,LEN)	A 686
	CALL TCATJ (SCT,STWK)	A 687
	CALL CPATJ (SCC,STWK)	A 688
	SARHO(KS)=ATJRH0	A 689
	SATC(KS)=SCT	A 690
	SACP(KS)=SCC	A 691
	IND(KS,LEN)=1	A 692
58	TW(KS,LEN)=TW(KS,LEN)+DELT/(SACP(KS)*SARHO(KS)*DE(KS,LEN))*(SHE*(S	A 693
	1TG -TW(KS,LEN))-SATC(KS)/DE(KS,LEN)*(TW(KS,LEN)-TW(KS+1,LEN)))**2.	A 694
	IF (TW(KS,LEN).GE.TMELT) GO TO 59	A 695
	JIND(LEN)=0	A 696
	GO TO 65	A 697
59	TW(KS,LEN)=TMELT	A 698
	HR=CPA*TE**2/(2.*G*AJ)	A 699
	SQCOND=-SATC(KS)*(TW(KS,LEN)-TW(KS+1,LEN))/DE(KS,LEN)	A 700
	SQTOT=SQDOTC+SQCOND	A 701
	IF (SQTOT.LE.0.) GO TO 65	A 702
	IND(KS,LEN)=1	A 703
	JIND(LEN)=1	A 704
	DELM=HR-CPA*TMELT	A 705
	QSTARS=2000.*2.*DELM	A 706
	CDOTS=SQTOT/(QSTARS*ATJRH0)	A 707
	SDOT(LEN)=SDOT(LEN)+CDOTS*DELT	A 708
	SS=CDOTS	A 709
	RX=RX+1./(1.-SIN(THETC))*(SIN(THETC)*SS-CDOT)	A 710
	SX=.01745*RX*PHI	A 711
	IF (SX.LE.0.) SX=.01943	A 712
	IF (RX.LE.0.) RX=.01387	A 713
	SNR=RX	A 714
	DO 60 N=1,NALS	A 715
	BN=N	A 716
	IF (BN*DEL(1,NOFXP)-SDOT(LEN)) 60,60,61	A 717
60	CONTINUE	A 718
	PRINT 84	A 719
	GO TO 69	A 720
61	DE(N,LEN)=DE(N,LEN)-(SDOT(LEN)-(BN-1.)*DEL(1,NOFXP))	A 721
	PCM=SACP(N)*SARHO(N)*DE(N,LEN)**2/(DELT*SATC(N))	A 722
	IF (PCM-2.) 62,63,63	A 723
62	DE(N,LEN)=0.	A 724
	SDOT(LEN)=BN*STOL	A 725
	IND(N,LEN)=1	A 726
	IND(N+1,LEN)=1	A 727
63	IF (N.EQ.1) GO TO 65	A 728
	NM=N-1	A 729
	DO 64 I=1,NM	A 730
64	DE(I,LEN)=0.	A 731
65	KS=KS+1	A 732
	IF (KS.LE.NALS) GO TO 56	A 733
	TW(KS,LEN)=TW(KS,LEN)+DELT*(SATC(KS-1)/(SARHO(KS-1)*SACP(KS-1)*DE(	A 734
	1KS-1,LEN)**2)*(TW(KS-1,LEN)-TW(KS,LEN))-SATC(KS)/(SARHO(KS)*SACP(K	A 735
	2S)*DE(KS,LEN)**2)*(TW(KS,LEN)-TW(KS+1,LEN)))	A 736
	IF (TW(KS,LEN).GT.1060.) TW(KS,LEN)=1060.	A 737
	KS=KS+1	A 738
	TW(KS,LEN)=TW(KS,LEN)+DELT*(SATC(KS-1)/(SARHO(KS-1)*SACP(KS-1)*DE(	A 739
	1KS-1,LEN)**2)*(TW(KS-1,LEN)-TW(KS,LEN)))	A 740
	IF (TW(KS,LEN).GT.860.) TW(KS,LEN)=860.	A 741
C	*****	A 742
	TMELT=6760.	A 743



C	*****	A 744
	TOTWT=0,	A 745
	WDOT=0.0	A 746
	DO 66 LL=1,NOFX	A 747
	TOTWT=TOTWT+PI*(RS(LL)*RL(LL)-SDOT(LL))*SDOT(LL)*RHOV*SOL(LL)	A 748
66	WDOT=WDOT+WT(LL)	A 749
	SWTL=.0698*PHI*SNRI**2*SDOT(NOFXP)*ATJRM0	A 750
	WT(NOFXP)=2.*PI*SNR**2*CDOTS*ATJRM0	A 751
	TOTWT=TOTWT+SWTL	A 752
	WDOT=WDOT+WT(NOFXP)	A 753
	VMA=BI-TOTWT/G	A 754
	RAD=RADI-SDOT(NOFXM)	A 755
	ALGTH=ALGTHI-SDOT(NOFXP)	A 756
	AREA=PI*RAD**2	A 757
	SLANT=ALGTH/(COS(THETC)-SNR/RAD*(1.-SIN(THETC)))	A 758
	ATL=AT-DELT/6.	A 759
	ATH=AT+DELT/6.	A 760
	IF (ALT.LE.0.0) GO TO 68	A 761
	IF (T.GT.ATL.AND.T.LT.ATH) GO TO 67	A 762
	GO TO 8	A 763
67	IF (LLL.LT.8) GO TO 68	A 764
	LLL=0	A 765
	PRINT 80	A 766
68	PRINT 81, T,X,Y,Z,ALT,GAMMA,VMA,PSI,V,OD,AMACH,CD,ANR,PHI,THE,ALP	A 767
	,BET,PS,ALPT,XE,YE,ZE,FD,FN,GFZ,CLAT,DLO,ABETA,GCIR,AIZ,AIX	A 768
	PRINT 82, CP,CAN,CPB,CDF,ANRE	A 769
	PRINT 83, RHO,VIS,AMACHE,TEMPE,RMOE,VE,VISG,PBE	A 770
	LLL=LLL+1	A 771
	AT=AT+1.0	A 772
	PRINT 87, (SDOT(I),I=1,NOFXP)	A 773
	PRINT 70, ((TW(I,J),J=1,NOFX),I=1,NP2)	A 774
	PRINT 86, TW(NALC1,NOFX),TW(NALC2,NOFX)	A 775
	PRINT 89, WDOT,TOTWT,RX	A 776
	PRINT 90, QSTAR	A 777
	PRINT 91, (TW(I,NOFXP),I=1,NP25)	A 778
	IF (ALT) 69,69,A	A 779
69	CONTINUE	A 780
C		A 781
C		A 782
70	FORMAT (30X,8F8.0)	A 783
71	FORMAT (1H1)	A 784
72	FORMAT (*JIND*10I4)	A 785
73	FORMAT (*IND*8I4/(10X,8I4))	A 786
74	FORMAT (*VCF*8E12.3/(11X,8E12.3))	A 787
75	FORMAT (*VTC*8E12.3/(11X,8E12.3))	A 788
76	FORMAT (*VRMO*8E12.3/(11X,8E12.3))	A 789
77	FORMAT (*DEL*8E12.3/(11X,8E12.3))	A 790
78	FORMAT (*DE*8E12.3/(11X,8E12.3))	A 791
79	FORMAT (*TW*8E12.3/(11X,8E12.3))	A 792
80	FORMAT (12X,4HTIME,14X,1HX,18X,1HY,15X,1HZ,16X,3HALT,13X,5HGAMMA,1	A 793
	11X,4HVMAS/30X,4HPSI,15X,3HVEL,13X,2HQD,15X,5HAMACH,11X,2HCD,14X,3H	A 794
	2REN/30X,3HPI,16X,5HTHETA,11X,5HALPHA,12X,3HBET,13X,2HPS,14X,4HALP	A 795
	3T/30X,2HXE,17X,2HYE,14X,2HZE,15X,2HFD,14X,2HFN,14X,3HGFZ/30X,3HLAT	A 796
	4,16X,4HLONG,12X,4HBETA,13X,4HGCIR,12X,2HIZ,14X,2HIX)	A 797
	FORMAT (1H0,4X,7E17.7/(22X,6E17.7))	A 798
81		A 799
82	FORMAT (10X,5E10.3)	A 800
83	FORMAT (5X,8E10.3)	A 801
84	FORMAT (10H OVERHEAT/)	A 802
85	FORMAT (*OVERHEATSTAG1*/)	A 803
86	FORMAT (*OVERHEATSTAG2*/)	A 804
87	FORMAT (10X*SDOT*9E12.3)	A 805
88	FORMAT (86X,8F8.0)	

89	FORMAT (10X*WTLOSSRATEIS*E11.4*LBS/SECANDTOTALWTLOSSIS*E13.4*LBSAN	A 806
	1DNOSERADIS*E11.4*FT*)	A 807
90	FORMAT (10X*QSTARIS*E11.4*BTU/LB*)	A 808
91	FORMAT (30X,13F8.0)	A 809
92	FORMAT(* SDOT* 10F8.5)	AB1C
	END	AB11

C	SUBROUTINE ATTK (VRZ4, VRY4, ALP, PI)	8	1
	COMPUTES ANGLE OF ATTACK, ALP.	8	2
1	IF (VRZ4) 9,5,1	8	3
2	IF (VRY4) 4,3,2	8	4
	ALP=ATAN(VRY4/VRZ4)	8	5
	RETURN	8	6
3	ALP=0.	8	7
	RETURN	8	8
4	ALP=ATAN(VRY4/VRZ4)	8	9
	RETURN	8	10
5	IF (VRY4) 6,7,8	8	11
6	ALP=-PI/2.	8	12
	RETURN	8	13
7	ALP=0.	8	14
	RETURN	8	15
8	ALP=PI/2.	8	16
	RETURN	8	17
9	IF (VRY4) 10,11,12	8	18
10	ALP=-PI+ATAN(VRY4/VRZ4)	8	19
	RETURN	8	20
11	ALP=PI	8	21
	RETURN	8	22
12	ALP=PI+ATAN(VRY4/VRZ4)	8	23
	RETURN	8	24
	END	8	25-

C	SUBROUTINE CDSF (CD,AM,ANR)	C	1
	COMPUTES INVISCID DRAG	C	2
	COMMON /B/ AREA,RAD,SLANT,SNR,THETC,V	C	3
	COMMON /C/ CPM,CP,CAN,CPR,CDF	C	4
	PI=3.14159	C	5
	G=32.174	C	6
	ANOSE=PI*(SNR**2)	C	7
	WAREA=PI*RAD*SLANT	C	8
	AMACH=AM	C	9
	PR=.89	C	10
	CPM=1.42857/(AM**2)*((6.*(AM**2)/5.)*3.5*(6./(7.*(AM**2)-1.))*2.	C	11
	15-1.)	C	12
	CP=4.*(SIN(THETC))*2*(2.5*8.*AMACH*SIN(THETC))/(1.*16.*AMACH*SIN(	C	13
	1THETC))	C	14
	CAN=(.55-CP)*ANOSE/AREA	C	15
	CPR=1.001/AM**2	C	16
	CALL CDFM (CDF)	C	17
	CD=CP+CAN+CPR+CDF	C	18
	RETURN	C	19
	END	C	20-

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C      SUBROUTINE LONG (DLG,XE,YE,PI)
      THIS SUB COMPUTES LONGITUDE
      IF (YE) 1,6,4
1     IF (XE) 2,9,3
2     DLG=-PI*ATAN(YE/XE)
      RETURN
3     DLG=ATAN(YE/XE)
      RETURN
4     IF (XE) 5,10,3
5     DLG=PI*ATAN(YE/XE)
      RETURN
6     IF (XE) 7,7,8
7     DLG=PI
      RETURN
8     DLG=0.0
      RETURN
9     DLG=-PI/2.
      RETURN
10    DLG=PI/2.
      RETURN
      END

```

```

0 1
0 2
0 3
0 4
0 5
0 6
0 7
0 8
0 9
0 10
0 11
0 12
0 13
0 14
0 15
0 16
0 17
0 18
0 19
0 20
0 21-

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	SUBROUTINE INITIAL (SNR,THETC,SLANT)	E	1
C		E	2
C	THIS SUBROUTINE PROVIDES THE NECESSARY GEOMETRY CALCULATIONS AND	E	3
C	CHANGES IN UNITS FOR PROGRAM STRAR6	E	4
C		E	5
	COMMON /D/ TW(50,12),IND(50,12),JIND(12),VCP(50,12),VTC(50,12),VRH	E	6
	10(50,12),DEL(50,12),DE(50,12),SDOT(12),XI(12),RS(12),RL(12),SOL(12	E	7
	2),WT(12),TM(12),TWT(12),SACP(50),SATC(50),SARNO(50),RN(12)	E	8
	COMMON /F/ NOFX,ALGTH,RADI,POS(11),AZN,GLAT	E	9
	AZN=AZN/57.296	E	10
	GLAT=GLAT/57.296	E	11
	ALGTH=POS(NCFX)/12.	E	12
	IM=NCFX-1	E	13
	STH=SIN(THETC)	E	14
	CTH=COS(THETC)	E	15
	DO 1 I=1,IM	E	16
	RN(I)=6.6E+07	E	17
	SOL(I)=(POS(I+1)-POS(I))/(12.0*CTH)	E	18
1	RS(I)=SNR*CTH*(POS(I)/12.0-SNR*(1.0-STH))*STH/CTH	E	19
	RN(NOFX)=5.0E+06	E	20
	RN(NOFX+1)=1.5E+06	E	21
	RS(NOFX)=SNR	E	22
	RS(NOFX+1)=SNR	E	23
	SOL(NOFX)=(POS(1)/12.0-SNR*(1.0-STH))/CTH	E	24
	RADI=SNR*CTH*(ALGTH-SNR*(1.0-STH))*STH/CTH	E	25
	IM=NCFX-2	E	26
	DO 2 I=1,IM	E	27
2	RL(I)=RS(I+1)	E	28
	RL(NOFX+1)=RADI	E	29
	RL(NOFX)=RS(1)	E	30
	SLANT=0.0	E	31
	DO 3 I=1,NOFX	E	32
3	SLANT=SLANT+SOL(I)	E	33
	RETURN	E	34
	END	E	35-

	SUBROUTINE CDFM (CDF)		
C	THIS SUR COMPUTES SKIN FRICTION DRAG	F	1
	COMMON /B/ AREA,RAD,SLANT,SNR,THETC,V	F	2
	COMMON /D/ TW(50,12),IND(50,12),JIND(12),VCP(50,12),VTC(50,12),VRH	F	3
	10(50,12),DEL(50,12),DE(50,12),SDOT(12),XI(12),RS(12),RL(12),SOL(12	F	4
	2),WT(12),TM(12),TWT(12),SACP(50),SATC(50),SARHO(50),RN(12)	F	5
	COMMON /E/ NOFXM,NAL,ALT,AMACH,QD,AMACHE,TEMPE,RHOE,VE,VISG,PBE,AN	F	6
	1RE	F	7
	TWA=0.	F	8
	CDF=0.0	F	9
	SR=SNR/RAD	F	10
	PI=3.14159	F	11
	CALL RHOF (RHO,ALT)	F	12
	CALL VISCO (VIS,ALT)	F	13
	CALL VSD (VS,ALT)	F	14
	CALL PRESS (PRE,ALT)	F	15
	CALL TEMPA (TEMP,ALT)	F	16
	AMACH=V/VS	F	17
	TEMPE=TEMP*(1+.0966*AMACH*SIN(THETC)+.2267*(AMACH*SIN(THETC))**2)	F	18
	VE=V*(1.-1.4/AMACH**2*(AMACH*SIN(THETC))**1.9)**.5	F	19
	AMACHE=AMACH*(VE/V)*(TEMP/TEMPE)**.5	F	20
	R1=1716.	F	21
	AM=AMACH	F	22
	AM=AMACH	F	23
	PBE=PRE*(1+.2.8*AM**2*(SIN(THETC))**2*(2.5+.8*AM*SIN(THETC)))/(1.+1	F	24
	16.*AM*SIN(THETC))	F	25
	RHOE=PBE/(R1*TEMPE)	F	26
	CALL VISCOT (VISG,TEMPE)	F	27
	AREA=PI*RAD**2	F	28
	ANRE=RHOE*VE*SLANT*32.174/VISG	F	29
	COTT=1./TAN(THETC)	F	30
	DO 2 K=1,NOFXM	F	31
	DO 1 M=1,NAL	F	32
	IF (DE(M,K).LE.0.) GO TO 1	F	33
	TWA=TWA+TW(M,K)/8.	F	34
1	CONTINUE	F	35
2	CONTINUE	F	36
	RE=RHO*V*SLANT/VIS	F	37
	IF (ANRE=2.0E+07) 3,4,4	F	38
3	HSN=.5+.5*(TWA/TEMPE)+.0374*(AMACHE)**2	F	39
	CDF1=(1.53/RE**5)*(VE/V)**1.5*(PBE/PRE)**.5*(TEMP/TEMPE)**.185*(1	F	40
	1./HSN)**.185*COTT	F	41
	CDF=CDF1*((1.+9.8842*SR-44.0235*SR**2+70.383*SR**3)-(SR**(.69+2.1*	F	42
	1SR)*ALOG10(RE))	F	43
	GO TO 5	F	44
4	HSN=.5+.5*(TWA/TEMPE)+.0388*(AMACHE)**2	F	45
	CDF1=(.0776/RE**2)*(VE/V)**1.8*(PBE/PRE)**.8*(TEMP/TEMPE)**.58*(1	F	46
	1./HSN)**.58*COTT	F	47
	CDF=CDF1*(1.-((.8+.052*AMACH)*SR))	F	48
5	CDF=CDF	F	49
	RETURN	F	50
	END	F	51
		F	52-

```

SUBROUTINE VISCOT (VISC,TEMP)
C THIS SUB COMPUTES VISCOSITY OF AIR VS TEMP
C UNITS ARE LB/FT-SEC
  IF (TEMP-2460.) 2,1,1
1  VISC=41.0E-10*TEMP+2.4414E-05
  RETURN
  IF (TEMP-1160.) 4,3,3
2  VISC=100.769E-10*TEMP+.97108E-05
3  RETURN
  IF (TEMP-760.) 6,5,5
4  VISC=132.5E-10*TEMP+.603E-05
5  RETURN
6  VISC=166.666E-10*TEMP+.34334E-05
  RETURN
END

```

```

G 1
G 2
G 3
G 4
G 5
G 6
G 7
G 8
G 9
G 10
G 11
G 12
G 13
G 14
G 15-

```



```

C  SUBROUTINE PRESS (PE,ALT)
C  COMPUTES PRESSURE PER ALTITUDE
C  LBS/FT2
  PE=2116.2*EXP(-4.25509E-05*ALT)
  RETURN
  END

```

```

H  1
H  2
H  3
H  4
H  5
H  6-

```

```

SUBROUTINE VSD (VS,ALT)
C THIS SUB COMPUTES SPEED OF SOUND
C UNITS ARE FT/SEC
IF (ALT-36000.) 1,1,2
1 VS=1116.9-.412E-02*ALT
RETURN
2 IF (ALT-82000.) 3,3,4
3 VS=968.5
RETURN
4 IF (ALT-154000.) 5,5,6
5 VS=814.+.191E-02*ALT
RETURN
6 IF (ALT-172000.) 7,7,8
7 VS=1106.
RETURN
8 IF (ALT-262000.) 9,9,10
9 VS=1527.-.245E-02*ALT
RETURN
10 VS=946.
RETURN
END

```

```

I 1
I 2
I 3
I 4
I 5
I 6
I 7
I 8
I 9
I 10
I 11
I 12
I 13
I 14
I 15
I 16
I 17
I 18
I 19
I 20
I 21-

```

```

SUBROUTINE GCPB (GCP,PBL)
C THIS SUB COMPUTES ABLATIVE GAS SPECIFIC HEAT
C UNITS ARE BTU/LR-DEG R
  IF (PBL-10.) 2,1,1
1  GCP=-.228*ALOG(PBL)+3.22
  RETURN
2  IF (PBL-.009) 4,3,3
3  GCP=-.30882*ALOG(PBL)+3.4
  RETURN
4  IF (PBL-.0001) 6,5,5
5  GCP=.7374*ALOG(PBL)+8.37
  RETURN
6  IF (PBL=1.0E-05) 8,7,7
7  GCP=.12333*ALOG(PBL)+2.709
  RETURN
8  GCP=1.284
  RETURN
END

```

```

J 1
J 2
J 3
J 4
J 5
J 6
J 7
J 8
J 9
J 10
J 11
J 12
J 13
J 14
J 15
J 16
J 17
J 18-

```

```

SUBROUTINE TEMPA (TEMP,ALT)
C THIS SUB COMPUTES TEMP OF AIR VS ALT TO 300,000
C TEMP IS IN DEGREES RANKINE
IF (ALT-261000.) 2,1,1
1 TEMP=298.2
  RETURN
2 IF (ALT-175500.) 4,3,3
3 TEMP=-2.463E-03*ALT+941.04
  RETURN
4 IF (ALT-154000.) 6,5,5
5 TEMP=508.79
  RETURN
6 IF (ALT-85000.) 8,7,7
7 TEMP=1.627E-03*ALT+256.05
  RETURN
8 IF (ALT-35700.) 10,9,9
9 TEMP=389.99
  RETURN
10 TEMP=-36.77E-04*ALT+518.7
  RETURN
END

```

```

K 1
K 2
K 3
K 4
K 5
K 6
K 7
K 8
K 9
K 10
K 11
K 12
K 13
K 14
K 15
K 16
K 17
K 18
K 19
K 20
K 21-

```

```

SUBROUTINE TCAT (TCA,TEMP)
C THIS SUB COMPUTES THERMAL COND. OF AIR VS. TEMP
C UNITS ARE BTU/SEC-FT-DEGREE RANKINE
  IF (TEMP-2960.) 2,1,1
1  TCA=6.0E-06*TEMP+.03324
  GO TO 9
2  IF (TEMP-2460.) 4,3,3
3  TCA=7.8E-06*TEMP+.028
  GO TO 9
4  IF (TEMP-1960.) 6,5,5
5  TCA=14.2E-06*TEMP+.01217
  GO TO 9
6  IF (TEMP-1460.) 8,7,7
7  TCA=16.5E-06*TEMP+.00766
  GO TO 9
8  TCA=19.28E-06*TEMP+.00444
9  TCA=TCA/3600.
  RETIJRN
END

```

```

L 1
L 2
L 3
L 4
L 5
L 6
L 7
L 8
L 9
L 10
L 11
L 12
L 13
L 14
L 15
L 16
L 17
L 18
L 19-

```

	SUBROUTINE VCPFC (VCP,TEMP)	M	1
C	THIS SUB COMPUTES VIRGIN SPECIFIC HEAT VS. TEMP	M	2
C	UNITS ARE BTU/LB-DEG R	M	3
	IF (TEMP-400.) 2,1,1	M	4
1	VCP=.55E-04*TEMP+1.23	M	5
	RETURN	M	6
2	IF (TEMP-2000.) 4,3,3	M	7
3	VCP=.19E-03*TEMP+.69	M	8
	RETURN	M	9
4	IF (TEMP-1060.) 6,5,5	M	10
5	VCP=.554E-03*TEMP-.0423	M	11
	RETURN	M	12
6	IF (TEMP-860.) 8,7,7	M	13
7	VCP=.975E-03*TEMP-.4865	M	14
	RETURN	M	15
8	IF (TEMP-660.) 10,9,9	M	16
9	VCP=.21E-03*TEMP+.1714	M	17
	RETURN	M	18
10	VCP=.375E-03*TEMP+.0625	M	19
	RETURN	M	20
	END	M	21-

	SUBROUTINE RHOF (RHO,ALT)	N	1
C	THIS SUB COMPUTES DENSITY OF AIR VS. ALTITUDE	N	2
	COMMON /A/ RE	N	3
	GALT=RE*ALT/(RE+ALT)	N	4
	RH00=.0023769	N	5
	R=53.352	N	6
	E=2.7183	N	7
	IF (GALT-36089.) 1,1,2	N	8
1	T=518.688-3.56616*GALT/1000.	N	9
	RHO=RH00*(T/518.668)**4.256	N	10
	RETURN	N	11
2	IF (GALT-82021.) 3,3,4	N	12
3	EP2=(36089.-GALT)/(R*389.988)	N	13
	RHO=RH00*.297069*E**EP2	N	14
	RETURN	N	15
4	IF (GALT-154199.) 5,5,6	N	16
5	T=254.988+1.44592*GALT/1000.	N	17
	RHO=RH00*.0326657*(T/389.988)**-12.388	N	18
	RETURN	N	19
6	IF (GALT-173885.) 7,7,8	N	20
7	EP4=(154199.-GALT)/(R*508.788)	N	21
	RHO=RH00*.00121179*E**EP4	N	22
	RETURN	N	23
8	IF (GALT-259186.) 9,9,10	N	24
9	T=938.088-2.46888*GALT/1000.	N	25
	RHO=RH00*5.86784E-04*(T/508.788)**6.592	N	26
	RETURN	N	27
10	IF (GALT-295276.) 11,11,12	N	28
11	EP6=(259186.-GALT)/(R*298.186)	N	29
	RHO=RH00*1.73274E-05*E**EP6	N	30
	RETURN	N	31
12	T=-349.412+2.19456*GALT/1000.	N	32
	RHO=RH00*1.792643E-06*(T/298.188)**-9.541	N	33
	RETURN	N	34
	END	N	35-

	SUBROUTINE VISCO (VIS,ALT)	0	1
C	THIS SUB COMPUTES VISCOSITY OF AIR VS. ALTITUDE	0	2
C	UNITS ARE IN LB-SEC/FT**2	0	3
	IF (ALT-270000.) 2.1,1	0	4
1	VIS=2.35E-07	0	5
	RETURN	0	6
2	IF (ALT-175500.) 4.3,3	0	7
3	VIS=-1.509E-12*ALT+6.333E-07	0	8
	RETURN	0	9
4	IF (ALT-156000.) 6.5,5	0	10
5	VIS=3.68E-07	0	11
	RETURN	0	12
6	IF (ALT-83250.) 8.7,7	0	13
7	VIS=.973E-12*ALT+2.172E-07	0	14
	RETURN	0	15
8	IF (ALT-34500.) 10.9,9	0	16
9	VIS=2.969E-07	0	17
	RETURN	0	18
10	VIS=-2.124E-12*ALT+3.737E-07	0	19
	RETURN	0	20
	END	0	21-



```

SUBROUTINE CCPP (CCP,TEMP)
C THIS SUB COMPUTES CHAR SPECIFIC HEAT VS. TEMP
C UNITS ARE BTU/LB-DEG R
  IF (TEMP-6000.) 2,1,1
1  CCP=.4
  RETURN
2  IF (TEMP-4000.) 4,3,3
3  CCP=.125E-04*TEMP+.3215
  RETURN
4  IF (TEMP-3000.) 6,5,5
5  CCP=.305E-04*TEMP+.2495
  RETURN
6  IF (TEMP-2000.) 8,7,7
7  CCP=.47E-04*TEMP+.14
  RETURN
8  IF (TEMP-1060.) 10,9,9
9  CCP=.207E-03*TEMP-.14
  RETURN
10 CCP=.96E-04*TEMP-.02236
  RETURN
END

```

```

P 1
P 2
P 3
P 4
P 5
P 6
P 7
P 8
P 9
P 10
P 11
P 12
P 13
P 14
P 15
P 16
P 17
P 18
P 19
P 20
P 21-

```

	SUBROUTINE SSLP (VRZ4,VRX4,BET,PI)	0	1
C	COMPUTES ANGLE OF SIDESLIP	0	2
	IF (VRZ4) 9.5,1	0	3
1	IF (VRX4) 4.3,2	0	4
2	BET=ATAN(VRX4/VRZ4)	0	5
	RETURN	0	6
3	BET=0.	0	7
	RETURN	0	8
4	BET=ATAN(VRX4/VRZ4)	0	9
	RETURN	0	10
5	IF (VRX4) 6.7,8	0	11
6	BET=-PI/2.	0	12
	RETURN	0	13
7	BET=0.	0	14
	RETURN	0	15
8	BET=PI/2.	0	16
	RETURN	0	17
9	IF (VRX4) 10.11,12	C	18
10	BET=-PI+ATAN(VRX4/VRZ4)	0	19
	RETURN	0	20
11	BET=PI	0	21
	RETURN	0	22
12	BET=PI+ATAN(VRX4/VRZ4)	0	23
	RETURN	0	24
	END	0	25-

```

C      SUBROUTINE VTCPC (VTC,TEMP)
      UNITS ARE BTU/FT-SEC-DEG R
      IF (TEMP-4000.) 2,1,1
1     VTC=.705E-07*TEMP+.58E-04
      RETURN
2     IF (TEMP-2000.) 4,3,3
3     VTC=.695E-07*TEMP+.62E-04
      RETURN
4     VTC=.7376E-07*TEMP+.5347E-04
      RETURN
      END

```

```

R      1
R      2
R      3
R      4
R      5
R      6
R      7
R      8
R      9
R     10
R     11-

```

```

SUBROUTINE ACMQ (C,A)
C      COMPUTES DAMPING COEFFICIENT
      IF (A.GT.17.) GO TO 1
      IF (A.GT.9.) GO TO 2
      IF (A.GT.7.) GO TO 3
      IF (A.GT.5.) GO TO 4
      IF (A.GT.4.) GO TO 5
      IF (A.GT.3.) GO TO 6
      C=2.5*A-10.
      RETURN
1      C=-2.4
      RETURN
2      C=.05*A-3.25
      RETURN
3      C=.1*A-3.7
      RETURN
4      C=.3*A-5.3
      RETURN
5      C=.7*A-7.3
      RETURN
6      C=A-8.5
      RETURN
      END

```

```

S      1
S      2
S      3
S      4
S      5
S      6
S      7
S      8
S      9
S     10
S     11
S     12
S     13
S     14
S     15
S     16
S     17
S     18
S     19
S     20
S     21
S     22
S     23-

```

C SUBROUTINE GASTC (GTC,GCP)  
 C COMPUTES ABLATIVE GAS THERMAL  
 C CONDUCTIVITY. BTU/SEC-FT-DEG.R.  
 GTC=4.429E-05\*(GCP+.297)  
 RETURN  
 END

T 1  
 T 2  
 T 3  
 T 4  
 T 5  
 T 6-

```

C      SUBROUTINE ACNA (C,A)
C      COMPUTES NORMAL FORCE COEFF.
C      CNA PER DEGREE
C      C=.034
C      RETURN
C      END

```

```

U      1
U      2
U      3
U      4
U      5
U      6-

```

```

SUBROUTINE CTCPC (CTC,TEMP)
C THIS SUB COMPUTES CHAR THERMAL CONDUCTIVITY VS. TEMP
C UNITS ARE BTU/FT-SEC-DEG R
IF (TEMP-6000.) 1,2,2
CTC=.56E-07*TEMP+.053E-03
RETURN
2 CTC=.389E-03
RETURN
END

```

```

V 1
V 2
V 3
V 4
V 5
V 6
V 7
V 8
V 9-

```

```

SUBROUTINE CPATJ (S,T)
C   COMPUTES SPEC HEAT OF ATJ GRAPHITE VS. TEMP
C   UNITS   BTU/LB-DEG-R
          IF (T-2000.) 1,2,2
1         IF (T-1000.) 4,3,3
2         S=3.482E-05*T+.35536
          RETURN
3         S=1.25E-04*T+.175
          RETURN
4         S=3.409E-04*T-.0409
          RETURN
END

```

```

W   1
W   2
W   3
W   4
W   5
W   6
W   7
W   8
W   9
W  10
W  11
W  12-

```



```

SUBROUTINE TCATJ (C,T)
C COMPUTES THER. COND. OF ATJ GRAPHITE VS. TEMP
IF (T-3460.) 1,3,3
1 IF (T-2460.) 2,4,4
2 IF (T-1460.) 5,5,5
3 C=(2.E-05*T+.2808)/60.
  RETURN
4 C=.35/60.
  RETURN
5 C=(-1.84E-04*T+.8026)/60.
  RETURN
6 C=(-3.82E-04*T+1.091)/60.
  RETURN
END

```

```

X 1
X 2
X 3
X 4
X 5
X 6
X 7
X 8
X 9
X 10
X 11
X 12
X 13
X 14-

```

AFWL-TR-68-61

APPENDIX II

STRAB-6 PROGRAM LISTING FOR MULTIPLE CASE RUN

WJM62,6,2500,65000.MOULDS,627A,MODE.  
 RUNJ(S,40000,,,,,77000)  
 RFL(40000)  
 STRAB62.

## PROGRAM STRAB62 (INPUT,OUTPUT)

C	6 DEGREE TRAJECTORY WITH ABLATION HEATING AND STAG	A	2
	COMMON /A/ RE	A	3
	COMMON /B/ AREA,RAD,SLANT,SNR,THETC,V	A	4
	COMMON /C/ CPM,CP,CAN,CPB,CDF	A	5
	COMMON /D/ TW(50,12),IND(50,12),JIND(12),VCP(50,12)VTC(50,12),VRH	A	6
	10(50,12),DEL(50,12),DE(50,12)SDOT(12),XI(12),RS(12),RL(12),SOL(12	A	7
	2),WT(12),TM(12),TWT(12),SACP(50),SATC(50),SARHO(50),RN(12),QSTR(12	A	8
	3),QDOTC(12),QTOT(12),QCOND(12)	A	9
	COMMON /E/ NOFXM,NAL,ALT,AMACH,QD,AMACHE,TEMPE,RHOE,VE,VISG,PBE,AN	A	10
	IRE	A	11
	COMMON /F/ NOFX,ALGTH,RADI,POS(11),AZN,GLAT	A	12
C	VEHICLE DATA	A	13
C	IF NOFX IS CHANGED CHANGE FORMAT 103	A	14
C	IF CONE ANGLE IS CHANGED SUB CDSF MUST BE CHANGED	A	15
700	1500 READ 1500,NCASE,(POS(J),J=1,8)		
	FORMAT(12,7A10,A8)		
	PRINT 1501,NCASE,(POS(J),J=1,8)		
	1501 FORMAT(1H1,30X,7A10,A8)		
	ETC.		

```

DATA SNR,THETC,BI,SM/.0208,.07853,6.27836,.59718/*      A 15
DATA P1,E,AJ,GRAV,C/3.14159,2.7183,778.,32.174,32.174/      A 17
DATA REQ,PE,ET,WE/20.927491E+06,.673852E-02,.08181333,0.7292E-04/      A 18
DATA NOFX,NAL,NALC,NALS/B,4,10,35/*      A 19
DATA DELT,DT,H,BT,AT/.05,.0005,.001,.05,1./      A 20
DATA AZN,GLAT,DION,GAMMA/154.,33.996,-107.5,-22.39/*      A 21
DATA V,Z,ALPHA/15167.,2.50E+05,7./*      A 22
DATA (POS(1),I=1,8)/6.607,18.570,30.533,42.496,54.459,66.422,78.38*      A 23
AREA=PI*RAD**2      A 757
SLANT=ALGTH/(COS(THETC)-SNR/RAD*(1.-SIN(THETC)))      A 758
ATL=AT-DELT/6.      A 759
ATH=AT+DELT/6.      A 760
IF (ALT.LE.0.0) GO TO 68      A 761
IF (T.GT.ATL.AND.T.LT.ATH) GO TO 67      A 762
GO TO 8      A 763
67 IF (LLJ.LT.8) GO TO 68      A 764
LLJ=0      A 765
PRINT 80      A 766
68 PRINT 81, T,X,Y,Z,ALT,GAMMA,VMA5,PSI,V,QD,ANACH,CD,ANR,PHI,THE,ALP      A 767
1,BET,PS,ALPT,XE,YE,ZE,FD,FN,GFZ,CLAT,DION,ABETA,GCIR,AIZ,AIX      A 768
PRINT 82, CP,CAN,CPB,CDF,ANRE      A 769
PRINT 83, RHO,VIS,AMACHE,TEMPE,RHOE,VE,VISC,PBE      A 770
LLJ=LLJ+1      A 771
AT=AT+1.0      A 772

```

PRINT 87, (SDOT(I), I=1, NOEXP)	A 773
PRINT 70, ((TW(I, J), J=1, NOFX), I=1, NP2)	A 774
PRINT 88, TW(NALC1, NOFX), TW(NALC2, NOFX)	A 775
PRINT 89, WDOT, TOTWT, RX	A 776
PRINT 90, QSTAR	A 777
PRINT 91, (TW(I, NOFX), I=1, NP2S)	A 778
IF (ALT) 69, 69, 8	A 779
IF (NCASE.NE.0) GO TO 700	A 780
	A 781
	A 782
FORMAT (30X, 8F8.0)	A 783
FORMAT (1H1)	A 784

69	
C	
C	
70	
71	

AFWL-TR-68-61

APPENDIX III  
SAMPLE CASE OUTPUT

## TIME

TIME	X PFI PHI XF LAT	Y VEL THETA YE LONG	Z OD ALPHA ZE BETA	ALT AMACH BET FO OCIR	GAMMA CO PS FN IZ	VHAS REN ALPT GFZ JX
0.	0.	0.	2.500000E+05	2.500000E+05	-2.239000E+01	6.278360E+00
	0.	1.516700E+04	8.908810E+00	1.658501E+01	1.394224E-01	3.347410E+04
	0.	0.	0.	0.	0.	7.000000E+00
	0.	0.	0.	0.	0.	0.
	3.381774E+01	-1.075000E+02	0.	0.	5.100000E-01	2.308000E+01
1.000000E+00	2.370650E-01	1.402363E+04	2.442076E+05	2.442134E+05	-2.249528E+01	6.278360E+00
	1.932204E-03	1.517858E+04	1.174093E+01	1.634430E+01	1.522152E-01	4.366317E+04
	1.071795E-02	-7.983113E-02	5.315783E+00	8.150359E-02	6.283162E-00	5.315783E+00
	1.757821E+07	6.147473E+03	1.176120E+07	1.929712E+00	2.291293E+00	-3.049192E-01
	3.378565E-01	-1.074799E+02	1.229026E+03	1.522376E+04	5.100000E-01	2.308000E+01
1.460E-02	6.739E-04	3.747E-03	1.034E-01	8.481E+04		
1.019E-07	1.648E-07	1.330E-01	5.081E+02	2.777E-07	1.511E+04	3.835E-05
SNUT 0.	0.	0.	0.	0.	0.	0.
	610	593	579	573	571	593
	546	545	545	545	545	585
	544	544	544	544	544	578
	544	544	544	544	544	572
	544	544	544	544	544	567
	544	544	544	544	544	563
						554
						545
WTLOSSRATEIS 0.	LBS/SECANDTOTALWTLOSSIS	0.	LBSANDNOSERADIS	2.0800E-02FT		
OSTARTIS 0.	BTU/LB	710	629	612	599	579
	553	551	548	547	546	545
	544	544	544	544	544	544
2.000000E+00	9.383426E-01	2.834730E+04	2.383850E+05	2.384046E+05	-2.260102E+01	6.278360E+00
	3.768125E-03	1.519015E+04	1.532187E+01	1.610996E+01	1.122113E-01	5.511221E+04
	1.120742E+02	-5.375531E-01	5.249595E-01	5.607865E-01	1.254590E-01	5.249595E-01
	1.758843E+07	1.229454E+04	1.174750E+07	1.856437E+00	2.952897E-01	-3.850317E-01
	3.375143E+01	-1.074599E+02	1.667180E+03	3.145968E+04	5.100000E-01	2.308000E+01
1.467E-02	6.739E-04	3.857E-03	9.080E-02	1.007E+05		
1.328E-07	2.735E-07	1.317E-01	5.252E+02	3.375E-07	1.513E+04	3.845E-05
SNUT 0.	0.	0.	0.	0.	0.	0.
	673	639	612	605	600	617
	550	549	547	547	547	608
	544	544	544	544	544	600
	544	544	544	544	544	593
	544	544	544	544	544	587
	544	544	544	544	544	582
						539
						548
WTLOSSRATEIS 0.	LBS/SECANDTOTALWTLOSSIS	0.	LBSANDNOSERADIS	2.0800E-02FT		
OSTARTIS 0.	BTU/LB	779	739	707	680	613
	569	565	557	555	551	550
	546	546	545	545	545	545
3.000000E+00	2.056897E+00	4.207097E+04	2.325319E+05	2.325750E+05	-2.270831E+01	6.278360E+00
						586
						580
						547
						544







1.482E-02 6.737E-04 4.555E-03 4.531E-02 3.008E-05  
 5.472E-07 3.240E-07 1.239E-01 6.254E-02 1.141E-06 1.519E-04 3.906E-05 1.224E+00 0. 0. 0. 0.

SHOT 0. 1011 906 649 812 787 768 757 758 740 724 711 700 690 665 608

WTLOSSRATEIS 0. LBS/SECANDTOTALWTLOSSIS 0. LBSANDNOSERADIS 2.0800E-02FT  
 OSTARIS 0. RTU/LB 1159 1061 984 923 874 834 801 772 748 727 709 692 673 655

9.0000000E+00 1.7789750E+01 1.2620338E+05 1.9076934E+05 1.9714832E+05 -2.3349328E+01 6.2783600E+00  
 1.4989356E-02 1.5270728E+04 7.8877520E+01 1.4627318E+01 6.4943661E-02 2.2990097E+05  
 1.1727372E-02 1.5733595E+00 1.6204045E+00 -1.5597048E+00 5.8544901E-01 1.6204045E+00  
 1.759266E-07 5.5309104E+04 1.1651083E+07 3.5312382E+00 4.8923173E+00 -3.9751216E-01  
 3.3511288E-01 -1.0731990E+02 2.8805986E+03 1.4568170E+05 5.1000000E-01 2.3080000E+01

1.485E-02 6.736E-04 4.078E-03 4.082E-02 3.643E+05  
 6.765E-07 3.358E-07 1.227E+01 6.419E+02 1.406E-06 1.521E+04 3.917E-05 1.551E+00 0. 0. 0.

SHOT 0. 1001 949 890 850 822 800 788 785 764 747 732 720 710 681 622

WTLOSSRATEIS 0. LBS/SECANDTOTALWTLOSSIS 0. LBSANDNOSERADIS 2.0800E-02FT  
 OSTARIS 0. RTU/LB 1244 1130 1041 970 914 868 830 799 772 749 728 711 695 593

1.0000000E+01 2.2179234E+01 1.4022331E+05 1.9070245E+05 1.9117014E+05 -2.3345048E+01 6.2783600E+00  
 2.0881793E-02 1.5281929E+04 9.7086730E+01 1.4435528E+01 6.0280828E-02 2.7536953E+05  
 1.1211028E-02 6.5057486E-01 1.3379386E+00 -6.3675739E-01 6.2627999E+01 1.3379386E+00  
 1.759266E-07 6.1637242E+07 6.3193270E+00 4.7687739E+00 -0.0767238E-01 2.3080000E+01  
 3.3476892E+01 -1.0729992E+02 3.1034183E+03 1.6182328E+05 5.1000000E-01 2.3080000E+01

1.488E-02 6.736E-04 4.803E-03 3.688E-02 4.424E+05  
 8.314E-07 3.448E-07 1.215E+01 6.581E+02 1.742E-06 1.522E+04 3.928E-05 1.947E+00 0. 0. 0.

SHOT 0. 1111 993 931 889 858 834 821 813 790 771 755 741 730 699 636

WTLOSSRATEIS 0. LBS/SECANDTOTALWTLOSSIS 0. LBSANDNOSERADIS 2.0800E-02FT  
 OSTARIS 0. RTU/LB 1341 1204 1102 1022 947 905 862 827 797 771 749 729 712 601

1.1000000E+01 2.7141567E+01 1.5424243E+05 1.8460552E+05 1.8517134E+05 -2.3559782E+01 6.2783600E+00  
 1.4671117E-02 1.5293069E+04 1.1884707E+02 1.4282424E+01 5.9491363E-02 3.2822690E+05  
 1.1607068E+02 -2.1130559E+00 -7.5018530E+00 2.1425745E+00 6.9110021E+01 2.5018530E+00  
 1.759275E+07 6.752472E+04 1.6223385E+07 7.6343865E+00 -1.0515935E+01 -4.7727204E-01  
 3.3442475E+01 -1.0727995E+02 3.1446013E+03 1.7817803E+05 5.1000000E-01 2.3080000E+01  
 1.492E-02 6.735E-04 4.931E-03 3.341E-02 5.383E+05 1.7817803E+05 5.1000000E-01  
 1.018E-06 3.539E-07 1.203E+01 6.747E+02 2.158E+06 1.523E+04 3.940E-05 2.498E+00 0. 0. 0.

WTLOSSRATEIS 0. BTU/LB LBS/SECANDTOTALWTLOSSIS 0. LBSANDHOSERADIS 2.0800E-02FT  
 OSTARSIS 0.

1.2000000E+01 3.1786459E+01 1.4826004E+05 1.784774E+05 1.7915075E+05 -2.3644901E+01 6.2783600E+00  
 2.2284899E-02 1.5304098E+04 1.447383E+02 1.4065223E+01 5.5970818E-02 3.8944601E+05  
 1.1127757E+02 2.5862578E+00 2.3753990E+00 -2.5791627E+00 7.5392619E+01 2.3753990E+00  
 1.7601230E+07 7.373342E+04 1.1609511E+07 8.7486103E+00 1.2622077E+01 -4.0619489E-01  
 3.3404038E+01 -1.0725998E+02 3.3419180E+03 1.9454488E+05 5.1000000E-01 2.3080000E+01  
 1.495E-02 6.735E-04 5.060E-03 3.034E-02 4.566E+05 1.7915075E+05 6.2783600E+00  
 1.236E-06 3.630E-07 1.191E+01 6.910E+02 2.679E+06 1.524E+04 3.932E-05 3.177E+00 0. 0. 0.

WTLOSSRATEIS 0. BTU/LB LBS/SECANDTOTALWTLOSSIS 0. LBSANDHOSERADIS 2.0800E-02FT  
 OSTARSIS 0.

1.3000000E+01 3.7894799E+01 1.8227671E+05 1.7231992E+05 1.7311017E+05 -2.3769558E+01 6.2783600E+00  
 2.387710E-02 1.5314961E+04 1.7734599E+02 1.3886334E+01 5.1984653E-02 4.7031510E+05  
 1.1577705E+02 -2.5629488E+00 -1.9942149E+00 2.5945809E+00 8.1675320E+01 1.9942149E+00  
 1.7503180E+07 7.9872485E+04 1.1395423E+07 9.9546844E+00 -1.2983843E+01 -4.8414961E-01  
 1.3371578E+01 -1.0724003E+02 3.5986844E+03 2.1092497E+05 5.1000000E-01 2.3080000E+01  
 1.499E-02 6.735E-04 4.101E-03 2.728E-02 8.190E+05 1.7311017E+05 6.2783600E+00  
 1.512E-06 3.680E-07 1.179E+01 6.992E+02 3.370E+06 1.525E+04 3.963E-05 4.043E+00 0. 0. 0.

WTLOSSRATEIS 0. RTU/LR LBS/SECANDTOTALWTLOSSIS 0. LBSANDNOSERADIS 2.0800E-02FT 753 683

OSTARIS 0.

1.4000000E+01 4.3347126E-01 1.9629149E+05 1.6613144E+05 1.0704501E+05 -2.3976507E+01 6.2783600E+00

2.599236E-02 1.5325611E+04 2.2123379E+02 1.3856791E+01 4.8235510E-02 5.8630253E+05

1.1204902E-00 2.6024628E+00 1.7731313E+00 -2.5902184E+00 8.7958310E+01 1.7731313E+00

1.7605094E-07 8.6011395E+04 1.1581716E+07 1.1522894E+01 1.4401474E+01 -4.3308828E-01

3.3339099E-01 -1.0722006E+02 3.8784004E+03 2.2731688E+05 5.1000000E-01 2.3080000E+01

1.498E-02 6.735E-04 5.213E-03 2.418E-02 1.060E+06

1.084E-06 3.680E-07 1.178E+01 6.985E+02 4.354E-06 1.526E+04 3.974E-05 5.219E+00 0. 0. 0.

5007 0.

WTLOSSRATEIS 0. RTU/LR LBS/SECANDTOTALWTLOSSIS 0. LBSANDNOSERADIS 2.0800E-02FT 753 683

OSTARIS 0.

1.5000000E+01 5.0261525E-01 2.1030477E+05 1.5991357E+05 1.6096582E+05 -2.3982305E+01 6.2783600E+00

2.4818769E-02 1.5335992E+04 2.7621350E+02 1.3866177E+01 4.4722456E-02 7.3150487E+05

1.1545582E-02 -2.1510748E+00 -1.4502537E+00 2.1832318E+00 9.4241271E+01 1.4502537E+00

1.7606982E-07 9.2148322E+04 1.1567795E+07 1.3338356E+01 -1.4706170E+01 -4.9568730E-01

3.3304599E-01 -1.0720014E+02 4.1830579E+03 2.4372149E+05 5.1000000E-01 2.3080000E+01

1.498E-02 6.735E-04 5.204E-03 2.144E-02 1.374E+06

2.149E-06 3.680E-07 1.178E+01 6.987E+02 5.644E-06 1.527E+04 3.986E-05 6.767E+00 0. 0. 0.

5007 0.

WTLOSSRATEIS 0. RTU/LR LBS/SECANDTOTALWTLOSSIS 0. LBSANDNOSERADIS 2.0800E-02FT 753 683

OSTARIS 0.

1.4000000E+01 4.3347126E-01 1.9629149E+05 1.6613144E+05 1.0704501E+05 -2.3976507E+01 6.2783600E+00

2.599236E-02 1.5325611E+04 2.2123379E+02 1.3856791E+01 4.8235510E-02 5.8630253E+05

1.1204902E-00 2.6024628E+00 1.7731313E+00 -2.5902184E+00 8.7958310E+01 1.7731313E+00

1.7605094E-07 8.6011395E+04 1.1581716E+07 1.1522894E+01 1.4401474E+01 -4.3308828E-01

3.3339099E-01 -1.0722006E+02 3.8784004E+03 2.2731688E+05 5.1000000E-01 2.3080000E+01

1.498E-02 6.735E-04 5.213E-03 2.418E-02 1.060E+06

1.084E-06 3.680E-07 1.178E+01 6.985E+02 4.354E-06 1.526E+04 3.974E-05 5.219E+00 0. 0. 0.

5007 0.

1.1368979E+02 2.0678500E+01 3.9423557E-01 -1.7378043E-01 1.0052448E+02 3.9423557E-01  
 1.7608845E+07 9.8284511E+04 1.1553686E+07 1.5095618E+01 5.0036138E-01 -4.7691382E-01  
 3.3270800E+01 -1.0718020E+02 4.6261300E+03 2.6013750E+05 5.1000000E-01 2.3080000E+01  
 1.498E-02 6.735E-04 5.199E-03 1.498E-02 1.783E+06  
 2.936E-06 3.679E-07 1.179E+01 6.990E-02 7.321E-04 1.528E+04 3.999E-05 8.780E+00 0. 0. 0.

3007 0. 1486 1207 1202 1142 1099 1066 1040 1041  
 728 698 682 672 664 658 655 650 989  
 586 578 575 572 570 568 568 940  
 552 550 549 549 548 548 915  
 545 545 545 544 544 544 867  
 544 544 544 544 544 544 813  
 735

WTLOSSRATEIS 0. LHS/SECANDTOTALWTLOSSIS 0. LBSANDNOSEERADIS 2.0800E-02FT

OSTARIS 0. RTU/LB 2239 1838 1594 1405 1273 1172 1091 1025 949 923 885 852 824  
 800 778 759 743 728 715 703 692 682 674 666 659  
 647 642 638 634 631 628 626 624 622 621 594

1.7000000E+01 6.3884691E+01 2.3832423E+05 1.4738711E+05 1.4873894E+05 -2.4193581E+01 6.2783600E+06  
 2.9740488E-02 1.5358325E+04 4.4209780E+02 1.3984111E+01 3.8544000E-02 1.1354423E+06  
 1.1324731E-02 2.3936144E+00 9.2965087E-01 -2.3733225E+00 1.0680889E-02 9.2985087E-01  
 1.7410801E+07 1.0441245E+05 1.1339907E+07 1.0409087E+01 1.5091845E+01 -4.8583327E-01  
 3.3235943E+01 -1.0718020E+02 4.6510681E+03 2.7856510E+05 5.1000000E-01 2.3080000E+01  
 1.498E-02 6.735E-04 5.119E-03 1.498E-02 2.412E+06

3.750E-06 3.619E-07 1.180E+01 6.867E-02 9.762E-06 1.529E+04 4.012E-05 1.150E+01 0. 0. 0.  
 9007 0. 1564 1364 1262 1197 1150 1114 1095 1099  
 746 712 695 684 675 669 665 1038  
 491 593 579 576 574 572 571 989  
 553 552 551 550 549 549 950  
 545 545 545 545 545 545 920  
 545 545 544 544 544 544 895  
 838 753

WTLOSSRATEIS 0. LHS/SECANDTOTALWTLOSSIS 0. LBSANDNOSEERADIS 2.0800E-02FT

OSTARIS 0. RTU/LB 2511 2002 1695 1489 1338 1224 1136 1063 1003 953 910 875 845  
 818 795 775 757 742 728 715 704 693 684 676 669  
 656 651 646 642 639 636 633 631 630 629 600

1.8000000E+01 7.1728240E+01 2.5329885E+05 1.4107951E+05 1.4259828E+05 -2.4299920E+01 6.2783600E+06  
 2.7031111E-02 1.5365015E+04 5.6859271E+02 1.4136101E+01 3.4902313E-02 1.5540608E+06  
 1.1428334E-02 7.7199140E+01 1.111727E-02 -7.4758812E-01 1.1308992E-02 1.111727E-02  
 1.7615492E+07 1.1052000E+05 1.1525941E+07 2.1430784E+01 2.3197698E-01 -5.1734083E-01  
 3.3200988E+01 -1.0718020E+02 5.1360006E+03 2.9300400E+05 5.1000000E-01 2.3080000E+01  
 1.498E-02 6.735E-04 5.004E-03 1.498E-02 3.244E+06

4.817E-06 3.559E-07 1.198E+01 6.766E-02 1.305E+05 1.530E+04 4.027E-05 1.515E+01 0. 0. 0.  
 9007 0. 1600 1440 1329 1258 1208 1169 1147 1107  
 764 728 709 696 687 680 676 1094  
 597 588 583 580 578 576 575 1036  
 555 553 552 551 550 550 550 991  
 546 545 545 545 545 545 955  
 545 545 544 544 544 544 926  
 858 773

WTLOSSRATEIS 0. LHS/SECANDTOTALWTLOSSIS 0. LBSANDNOSEERADIS 2.0800E-02FT

OSTARIS 0. RTU/LB

2800 2209 1820 1594 1411 1202 1183 1104 1038 983 937 899 866  
837 791 772 756 741 727 715 704 686 678 671  
655 651 647 644 641 639 637 636 636 636 636

1.0000000E+01 7.9897015E+01 2.6633190E+05 1.3747244E+05 1.3643140E+05 -2.4404908E+01 6.2783600E+00  
2.7930974E+02 1.5373523E+04 7.3591308E+02 1.4308407E+01 3.3113303E+02 2.0444751E+06  
1.1464807E+02 -2.5029848E+02 -2.4267724E+01 5.2375482E+02 1.1937268E+02 2.4267724E+01  
1.7014277E+07 1.1608289E+05 1.1511960E+07 2.6312421E+01 -0.5504120E+00 -5.4730140E+01  
3.3166417E+01 -1.0712046E+02 5.6495907E+03 3.0945319E+05 5.1000000E+01 2.3080000E+01

1.491E-02 6.730E-04 4.891E-03 1.234E-02 4.423E+06 0.0000000E+00 0.0000000E+00  
6.227E-06 3.499E-07 1.207E+01 6.664E+02 1.747E-05 1.531E+04 4.045E-05 1.998E+01 0.0000000E+00

1760 1523 1403 1327 1271 1229 1205 1246  
744 723 710 700 693 688 1157  
604 594 588 582 580 579 1089  
557 555 553 552 551 551 1034  
546 545 545 545 545 545 994  
545 545 545 545 545 545 961  
545 545 545 545 545 545 882  
545 545 545 545 545 545 793

WTLOSSRATEIS O. ATU/LR LBS/SECANDTOTALWTLOSSIS 0. LBSANDNOSERADIS 2.0800E-02FT

3132 2457 1982 1690 1492 1344 1233 1146 1075 1016 965 923 888  
857 831 808 788 770 754 740 727 716 705 696 688 681  
674 669 663 659 655 652 649 647 645 644 644 644 614

2.0000000E+01 8.8287010E+01 2.8032964E+05 1.2837622E+05 1.3024791E+05 -2.4510096E+01 6.2783600E+00  
2.9815136E+02 1.5381222E+04 9.5840964E+02 1.4472719E+01 3.0976847E+02 2.7076847E+06  
1.1452332E+02 9.2565999E+01 -2.0610044E+02 -8.9833741E+01 1.2565940E+02 2.0610044E+02  
1.7610358E+07 1.2281148E+05 1.1497967E+07 3.2084798E+01 -7.2517155E+01 -5.7375907E+01  
3.3131322E+01 -1.0710058E+02 6.0392402E+03 3.2591181E+05 5.1000000E+01 2.3080000E+01

1.480E-02 6.730E-04 4.779E-03 1.081E-02 6.004E+06 0.0000000E+00 0.0000000E+00  
1.102E-06 3.499E-07 1.217E+01 6.561E+02 2.342E-05 1.532E+04 4.046E-05 2.637E+01 0.0000000E+00

1871 1614 1485 1402 1342 1296 1270 1337  
804 761 739 725 714 706 701 1220  
611 599 593 589 586 584 583 1146  
559 556 555 554 553 553 1006  
546 546 546 546 545 545 1037  
545 545 545 545 545 545 999  
545 545 545 545 545 545 908  
545 545 545 545 545 545 814

WTLOSSRATEIS O. ATU/LR LBS/SECANDTOTALWTLOSSIS 0. LBSANDNOSERADIS 2.0800E-02FT

3514 2732 2172 1814 1501 1414 1288 1191 1114 1049 995 949 913  
878 849 825 803 784 767 752 739 727 716 707 698 690  
684 678 672 668 663 660 657 655 653 652 652 652 621

2.1000000E+01 9.7080557E+01 2.9432216E+05 1.2198129E+05 1.2404503E+05 -2.4614451E+01 6.2783600E+00  
3.9469349E+02 1.5387623E+04 9.5840964E+02 1.4472719E+01 3.0976847E+02 2.7076847E+06  
1.1437110E+02 9.2565999E+01 -2.0610044E+02 -8.9833741E+01 1.2565940E+02 2.0610044E+02  
1.7617767E+07 1.2281148E+05 1.1497967E+07 3.2084798E+01 -7.2517155E+01 -5.7375907E+01  
3.3131322E+01 -1.0710058E+02 6.0392402E+03 3.2591181E+05 5.1000000E+01 2.3080000E+01

1.480E-02 6.730E-04 4.669E-03 9.052E-03 6.163E+06 0.0000000E+00 0.0000000E+00  
1.061E-05 3.379E-07 1.228E+01 6.450E+02 2.342E-05 1.532E+04 4.046E-05 2.637E+01 0.0000000E+00

1991 1715 1575 1485 1420 1370 1341 1443  
826 780 756 740 729 720 715 1309  
616 605 599 595 591 589 587 1213

WTLOSSRATEIS O. ATU/LR LBS/SECANDTOTALWTLOSSIS 0. LBSANDNOSERADIS 2.0800E-02FT

WTLOSSRATEIS 0.  
OSTARTIS 0.

WTU/LB  
3926 3042 2403 1960 1662 1491 1348 1240 1154 1085 1026 976 934

LBS/SECANDTOTALWTLOSSIS 0.  
LBSANDNOSERADIS 2.0800E-02FT

2.2000000E+01  
1.482E-02 6.737E-04 4.561E-03 7.757E-03 1.112E-07 6.7197289E-03 3.5685366E-05 5.1000000E-01 2.3080000E+01

1.400E-05 3.318E-07 1.239E-01 6.351E-02 4.22E-05 1.533E-04 4.118E-05 4.609E-01 0. 0. 0. 0. 0.

1.0674528E-02 3.0830839E-05 1.1555825E-05 1.1782338E-05 -2.4718546E-01 6.2783600E+00

4.0567211E-02 1.5393041E-04 1.9583800E-03 1.7614638E-01 2.7839914E-02 4.852566E+06

1.1477279E-02 -1.6840097E-03 -3.8427284E-02 1.7212335E-00 1.3822042E-02 3.8427284E-02

1.7619473E-07 1.3508043E-05 1.4669945E-07 4.9822307E-01 -2.3395596E+00 -6.6581305E-01

3.3062627E-01 -1.0706091E-02 7.757E-03 1.112E-07 6.7197289E-03 3.5685366E-05 5.1000000E-01 2.3080000E+01

1.482E-02 6.737E-04 4.561E-03 7.757E-03 1.112E-07 6.7197289E-03 3.5685366E-05 5.1000000E-01 2.3080000E+01

1.400E-05 3.318E-07 1.239E-01 6.351E-02 4.22E-05 1.533E-04 4.118E-05 4.609E-01 0. 0. 0. 0. 0.

1.0674528E-02 3.0830839E-05 1.1555825E-05 1.1782338E-05 -2.4718546E-01 6.2783600E+00

4.0567211E-02 1.5393041E-04 1.9583800E-03 1.7614638E-01 2.7839914E-02 4.852566E+06

1.1477279E-02 -1.6840097E-03 -3.8427284E-02 1.7212335E-00 1.3822042E-02 3.8427284E-02

1.7619473E-07 1.3508043E-05 1.4669945E-07 4.9822307E-01 -2.3395596E+00 -6.6581305E-01

3.3062627E-01 -1.0706091E-02 7.757E-03 1.112E-07 6.7197289E-03 3.5685366E-05 5.1000000E-01 2.3080000E+01

1.482E-02 6.737E-04 4.561E-03 7.757E-03 1.112E-07 6.7197289E-03 3.5685366E-05 5.1000000E-01 2.3080000E+01

1.400E-05 3.318E-07 1.239E-01 6.351E-02 4.22E-05 1.533E-04 4.118E-05 4.609E-01 0. 0. 0. 0. 0.

1.0674528E-02 3.0830839E-05 1.1555825E-05 1.1782338E-05 -2.4718546E-01 6.2783600E+00

4.0567211E-02 1.5393041E-04 1.9583800E-03 1.7614638E-01 2.7839914E-02 4.852566E+06

1.1477279E-02 -1.6840097E-03 -3.8427284E-02 1.7212335E-00 1.3822042E-02 3.8427284E-02

1.7619473E-07 1.3508043E-05 1.4669945E-07 4.9822307E-01 -2.3395596E+00 -6.6581305E-01

3.3062627E-01 -1.0706091E-02 7.757E-03 1.112E-07 6.7197289E-03 3.5685366E-05 5.1000000E-01 2.3080000E+01

1.482E-02 6.737E-04 4.561E-03 7.757E-03 1.112E-07 6.7197289E-03 3.5685366E-05 5.1000000E-01 2.3080000E+01

1.400E-05 3.318E-07 1.239E-01 6.351E-02 4.22E-05 1.533E-04 4.118E-05 4.609E-01 0. 0. 0. 0. 0.

1.0674528E-02 3.0830839E-05 1.1555825E-05 1.1782338E-05 -2.4718546E-01 6.2783600E+00

4.0567211E-02 1.5393041E-04 1.9583800E-03 1.7614638E-01 2.7839914E-02 4.852566E+06

1.1477279E-02 -1.6840097E-03 -3.8427284E-02 1.7212335E-00 1.3822042E-02 3.8427284E-02







WTLOSSRATEIS 9.7374E-04LBS/SECANDTOTALWTLOSSIS 1.0924E-03LBSANDNOSERADIS 2.7088E-02FT  
 QSTARIS 0.  
 RTU/LB  
 690 669 618 651 645 640 638 2047  
 584 578 575 572 571 569 568 1777  
 552 551 550 549 549 547 547 1454  
 549 548 548 548 547 547 547 1060  
 860

3.0000000E+01 1.9415298E+02 4.1937260E+05 6.3431662E+04 6.7638309E+04 -2.5561643E+01 6.2782796E+00  
 4.5493393E-02 1.5154919E+04 1.7967425E+04 1.5647826E+01 3.3701085E-02 6.0622161E+07  
 1.1555821E-02 -6.3148071E-02 4.6528959E-03 1.0414372E-01 1.884454E+02 4.6528959E-03  
 1.7632097E-07 1.8367024E+05 1.1358009E+07 6.5455272E+02 3.0725785E+00 -3.6718282E+00  
 3.2786897E+01 -1.0690318E+02 5.5509858E+03 4.9020732E+05 5.1000000E-01 2.3080000E+01  
 1.469E-02 1.435E-03 4.088E-03 1.347E-02 1.220E+08  
 1.566E-04 2.969E-07 1.289E+01 5.695E+02 4.284E-04 1.509E+04 4.597E-03 4.107E+02  
 SNOT 0. 0. 0. 0. 0. 0. 0. 8.742E-03

WTLOSSRATEIS 9.6317E-04LBS/SECANDTOTALWTLOSSIS 2.5854E-03LBSANDNOSERADIS 3.0470E-02FT  
 QSTARIS 0.  
 RTU/LB  
 600 640 5156 4176 3394 2767 2281 1935 1698 1526 1394 1290 1208  
 1140 1082 1034 992 955 924 898 874 854 836 820 807 794  
 784 774 766 759 752 747 743 739 737 735

3.1000000E+01 2.0650734E+02 4.3299196E+05 5.6907058E+04 6.1386296E+04 -2.5673637E+01 6.2782652E+00  
 5.4745747E-02 1.5048256E+04 2.3907215E+04 1.5537694E+01 3.2987327E-02 8.1484728E+07  
 1.1583447E-02 3.2928606E-01 -1.6381869E-01 -2.7816731E-01 1.9476779E-01 1.6381869E-01  
 1.7633330E+07 1.8962960E+05 1.1344205E+07 8.5154908E+02 -1.4378213E+02 -4.6514248E+00  
 3.2752972E+01 -1.0688387E+02 5.6710814E+03 5.0638062E+05 5.1000000E-01 2.3080000E+01  
 1.471E-02 1.694E-03 4.146E-03 1.222E-02 1.504E+08  
 2.111E-04 2.969E-07 1.283F+01 5.673E+02 5.500E-04 1.498E+04 4.722E-05 5.412E+02  
 SNOT 0. 0. 0. 0. 0. 0. 0. 1.031E-02

WTLOSSRATEIS 1.7771E-03LBS/SECANDTOTALWTLOSSIS 3.0498E-03LBSANDNOSERADIS 3.3143E-02FT  
 QSTARIS 0.  
 RTU/LB  
 6400 6400 5557 4481 3631 2957 2438 2044 1780 1591 1447 1334 1245  
 1172 1111 1059 1015 976 943 914 890 868 850 833 819 806  
 794 785 776 768 762 757 752 748 746 744 746 744 744  
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 ALPHA  
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 HETA  
 ALT  
 AMACH  
 BET  
 FU  
 GCIR  
 GAMMA  
 CD  
 PS  
 FN  
 IX

3.2000000E+01	2.1914943E+02	4.4648001E+05	5.0404294E+04	5.5168526E+04	-2.5782704E+01	6.2782479E+00
	5.7901008E-02	1.4907094E+04	3.1579669E+04	1.5391940E+01	3.2028597E-02	1.0920181E+08
	1.157786E-02	8.3928392E-02	3.6701442E-03	-3.1833118E-02	2.0105108E-02	3.6701442E-03
	1.7634924E+07	1.9553462E+05	1.1330507E-07	1.0921373E+03	4.2550177E+00	-5.8416131E+00
	3.2719328E+01	-1.0686473E+02	5.8408212E+03	5.2242198E+05	5.1000000E-01	2.3080000E+01
1.473E-02	2.038E-03	4.225E-03	1.103E-02	2.047E+08		
2.842E-04	2.969E-07	1.274E+01	5.844E+02	1.191E-04	1.484E+04	4.952E-05
SNOT 0.	0.	0.	0.	0.	0.	1.220E-02
	4446	3694	4222	4804	5029	5138
	1401	1206	1132	1136	1153	1182
	729	700	686	678	673	668
	596	588	584	581	579	577
	556	554	553	552	551	551
	552	550	550	549	549	549
						1643
						1600
						860
WTLOSSRATEIS 2.1239E-03LBS/SECANDTOTALLWTLOSSIS	3.6074E-03LBSANDNOSERADIS 3.6353E-02FT					
OSTARIS 0.	RTU/LR					
	6400	6400	5947	4800	3849	3166
	1205	1140	1085	1038	997	962
	806	795	786	778	772	766
						761
						758
						755
						753
						716
						1848
						1859
						1503
						1380
						1283
						831
						817
3.3000000E+01	2.3200755E+02	4.5982552E+05	4.3946807E+04	4.9001293E+04	-2.5891421E+01	6.2781990E+00
	5.8101429E-02	1.4721625E+04	4.1364330E+04	1.5200439E+01	3.1563017E-02	1.4570261E+08
	1.1585339E-02	2.6566337E-02	3.8380305E-02	2.5293350E-02	2.0733441E+02	3.8380305E-02
	1.7634270E+07	2.0136999E+05	1.1316953E-07	1.4097208E+03	5.8283006E+01	-7.4150638E+00
	3.2684055E+01	-1.0684583E+02	5.9269317E+03	5.3828916E+05	5.1000000E-01	2.3080000E+01
1.476E-02	2.512E-03	4.332E-03	9.917E-02	2.631E+08		
3.817E-04	2.949E-07	1.262E+01	5.606E+02	9.259E-04	1.466E+04	5.092E-05
SNOT 0.	0.	0.	0.	0.	0.	1.752E-02
	4747	4159	5384	5730	5849	5901
	1501	1279	1262	1293	1320	1352
	745	713	698	691	687	685
	601	592	588	585	583	581
	557	553	554	553	552	552
	553	551	551	550	550	549
						1740
						1060
						860
WTLOSSRATEIS 2.2213E-03LBS/SECANDTOTALLWTLOSSIS	5.1812E-03LBSANDNOSERADIS 4.0316E-02FT					
OSTARIS 0.	RTU/LR					
	6400	6400	6400	5174	4167	3388
	1239	1170	1112	1062	1019	962
	817	806	797	788	781	776
						771
						767
						764
						762
						1732
						1561
						1428
						1323
						843
						829
3.4000000E+01	2.4497912E+02	4.7295956E+05	3.7556386E+04	4.2905430E+04	-2.6001680E+01	6.2781758E+00
	5.8331901E-02	1.4490170E+04	5.3572294E+04	1.4951131E+01	3.1170063E-02	1.9291094E+08
	1.1594520E-02	2.2035493E-02	3.6884319E-02	3.0004922E-02	2.1361776E-02	3.6884319E-02
	1.7637582E+07	2.0711604E+05	1.1303580E+07	1.8030579E+03	7.2542527E+01	-9.3641090E+00
	3.2651266E+01	-1.0682721E+02	6.0016335E+03	5.5392817E+05	5.1000000E-01	2.3080000E+01
1.480E-02	2.959E-03	4.478E-03	8.893E-03	3.351E+08		
5.1110E-04	2.969E-07	1.247E+01	5.558E+02	1.185E-02	1.442E+04	5.201E-05
SNOT 0.	0.	0.	0.	0.	0.	1.955E-02
	5044	5665	6386	6566	6805	6802
	1613	1409	1452	1494	1528	1550
	763	727	714	708	705	702
	606	597	592	587	585	584
	558	556	555	554	553	553
	554	552	551	551	550	550
						6400
						4867
						3710
						2859
						2258
						1860

WTLOSSRATEIS 5.6249E-03LBS/SECANDTOTALWTLOSSIS 5.9280E-03LBSANDNOSERADIS 4.3532E-02FT  
 QSTARIS 6.2522E-03HTU/LB

6400	6400	6400	5660	4521	3650	2975	2462	2072	1811	1622	1479	1365
1275	1201	1139	1087	1042	1002	968	939	914	892	873	856	841
828	817	807	799	791	785	780	776	773	771	773		

3.5000000E+01 2.5801741E+02 4.8583481E+05 3.1260808E+04 3.6906678E+04 -2.6114147E+01 6.2421419E+00  
 5.8779700E-02 1.4172297E+04 6.8392552E+04 1.4633244E+01 3.0227403E-02 2.5000087E+08  
 1.1609253E-02 3.3366191E-02 2.1895502E-02 1.9185201E-02 2.1990112E-02 2.1895502E-02  
 1.7638830E+07 2.1274858E+05 1.1290489E+07 2.252712E+03 5.480445E+01 -1.1514961E+01  
 3.2621099E+01 -1.0680897E-02 6.1753212E+03 5.6927256E+05 5.1000000E-01 2.3080000E+01  
 1.485E-02 2.488E-03 4.674E-03 8.190E-03 4.171E+08 6.810E-04 2.969E-07 1.227E+01 5.498E+02 1.505E-03 1.419E+03  
 5DOT 0. 1.573E-04 9.693E-04 1.121E-03 1.130E-03 1.080E-03 9.860E-04 4.148E-04 2.234E-02

WTLOSSRATEIS 1.4796E-03LBS/SECANDTOTALWTLOSSIS 1.1653E-03LBSANDNOSERADIS 3.9515E-02FT  
 QSTARIS 5.8218E-03HTU/LB

6400	6400	6400	6182	4918	3950	3204	2634	2198	1899	1689	1532	1410
1312	1233	1167	1112	1064	1023	987	956	930	906	886	869	853
840	828	818	809	801	795	790	786	782	780	742		

3.6000000E+01 2.7104423E+02 4.9839173E+05 2.5089733E+04 3.1032908E+04 -2.0230124E+01 6.1844773E+00  
 6.0004137E-02 1.3801142E+04 8.1666354E+04 1.3954017E+01 2.9191509E-02 2.8949270E+08  
 1.1622653E-02 5.088753E-02 3.7060172E-03 3.7001607E-03 2.2618448E+02 3.7060172E-03  
 1.7840033E+07 2.1624158E+05 1.127657E+07 2.553016E+03 1.1022140E+01 -1.3268859E+01  
 3.2589795E+01 -1.0679118E-02 6.3667841E+03 5.8425059E+05 5.1000000E-01 2.3080000E+01  
 1.497E-02 1.199E-03 5.140E-03 7.880E-03 4.700E+08 8.575E-04 3.078E-07 1.184E+01 5.573E+02 1.794E-03 1.374E+04 5.253E-05 1.717E+03  
 5DOT 0. 2.092E-03 2.702E-03 2.745E-03 2.873E-03 2.559E-03 2.428E-03 1.215E-03 2.868E-02

WTLOSSRATEIS 1.5392E-03LBS/SECANDTOTALWTLOSSIS 3.0206E-03LBSANDNOSERADIS 2.6956E-02FT  
 QSTARIS 5.4828E-03HTU/LB

6400	6400	6400	6400	5647	4390	3495	2835	2348	2001	1762	1590	1457
1351	1266	1196	1138	1088	1044	1007	974	946	922	900	882	866
852	839	829	819	812	805	800	795	792	789	751		

3.7000000E+01 2.8398555E+02 5.1057803E+05 1.9669689E+04 2.5308779E+04 -2.6349855E+01 6.1251836E+00  
 6.1514541E-02 1.3373414E+04 9.4332156E+04 1.3206643E+01 2.8715926E-02 3.2350201E+08  
 1.1634400E-02 5.393236E-02 1.5451962E-03 1.1685716E-03 2.3246784E-02 3.7060172E-03  
 1.7641169E+07 2.2357218E+05 1.1265206E+07 2.8866867E+03 5.2812623E+00 -1.5084538E+01  
 3.2559215E+01 -1.0677391E-02 6.4431403E+03 5.9879891E+05 5.1000000E-01 2.3080000E+01  
 1.511E-02 2.456E-04 5.739E-03 7.620E-03 5.067E+08 1.055E-03 3.199E-07 1.135E+01 5.719E+02 2.090E-03 1.332E+04 5.183E-05 2.050E+03  
 1.055E-03 3.199E-07 1.135E+01 5.719E+02 2.090E-03 1.332E+04 5.183E-05 2.050E+03

SDOT	0.	4.048E-03	4.485E-03	4.423E-03	4.270E-03	4.091E-03	3.920E-03	2.274E-03	3.682E-02	
		6527	6760	6760	6760	6760	6760			
		2016	4168	4770	4733	4658	4562			
		826	859	892	901	897	890			
		625	615	609	608	606	605			
		563	560	559	557	556	556			
		557	555	554	553	552	552			
										860
										1060
										1528
										878
										1566
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										815
										619
										825
										632
										620



WTLOSSRATEIS	A.1076E-01LB5/SEC	TOTALWTLOSSIS	1.4983E-01LB5ANDNOSERADIS	7.4634E-03FT
OSTARIS 3.2275E+03RTU/LB	6400	6400	6400	6400
	6400	6400	4220	1739
	439	919	899	869
			878	861
			855	856
			860	856
			861	856
			862	856
			863	856
			864	856
			865	856
			866	856
			867	856
			868	856
			869	856
			870	856
			871	856
			872	856
			873	856
			874	856
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			876	856
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			891	856
			892	856
			893	856
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			896	856
			897	856
			898	856
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			900	856
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			902	856
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DOCUMENT CONTROL DATA - R & D		
<i>(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)</i>		
1. ORIGINATING ACTIVITY (Corporate author) Air Force Weapons Laboratory (WLDE) Kirtland Air Force Base, New Mexico 87117		2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED
		2b. GROUP
3. REPORT TITLE  AN INTEGRATED SIX-DEGREE-OF-FREEDOM TRAJECTORY AND AERODYNAMIC HEATING AND ABLATION COMPUTER PROGRAM (STRAB-6)		
4. DESCRIPTIVE NOTES (Type of report and inclusive dates) July 1966-November 1967		
5. AUTHOR(S) (First name, middle initial, last name) W. J. Moulds		
6. REPORT DATE August 1968	7a. TOTAL NO. OF PAGES 162	7b. NO. OF REFS 36
8a. CONTRACT OR GRANT NO.		9a. ORIGINATOR'S REPORT NUMBER(S) AFWL-TR-68-61
b. PROJECT NO. 5791		9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)
c. Task No. 579122		
d.		
10. DISTRIBUTION STATEMENT This document is subject to special export controls and each transmittal to foreign governments or foreign nationals may be made only with prior approval of AFWL (WLDE), Kirtland AFB, NM, 87117. Distribution is limited because of the technology discussed in the report.		
11. SUPPLEMENTARY NOTES		12. SPONSORING MILITARY ACTIVITY AFWL (WLDE) Kirtland AFB, NM 87117
13. ABSTRACT (Distribution Limitation Statement No. 2) An integrated computer program (STRAB-6) for the complete analysis of a blunt, conical reentry vehicle as to its trajectory and aerothermal environment upon atmospheric reentry is presented. The trajectory portion of the program calculates the vehicle motions in six-degrees-of-freedom, while the thermal portion calculates the aerodynamic heating and ablation of the vehicle. The earth model is an oblate, rotating spheroid whose parameters and gravitational potential are described herein. The equations of motion, the method of integration, thermal model and input forms for the program are discussed. The thermal model examined in this report assumes thick skin solution where the skin is of any composite structure made of discrete layers of material whose properties may vary from layer to layer. Also, the heatshield is comprised of one, two, or more different ablative materials. STRAB-6 computes nose-blunting and includes this effect in the aerodynamics. STRAB-6 is written in FORTRAN IV for the CDC 6600 computer.		

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		ROLE	WT	ROLE	WT	ROLE	WT
	Six-degrees of freedom Equations of motion Euler angles Moment of momentum Runge-Kutta Aerodynamic heating Ablation Heat transfer Boundary layer Hypersonic Reentry vehicles						

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